A Forest Fire Theory of the Duration of a Boom and the Size of a Subsequent Bust*

Matthew O. Jackson† and Pietro Tebaldi‡

Draft: February 2019

Abstract

We show that, in a large class of models, market frictions lead to predictable dynamic patterns of the acquisition and subsequent shedding of inputs by firms. The logic is as follows. During high demand and expansionary periods, firms that fail to have inputs (machinery, labor, space, credit) in place forego potentially large profit opportunities. Frictions in searching for inputs thus lead firms to accept sub-optimal inputs to take advantage of current profit opportunities. The longer an expansionary period lasts, the greater the buildup in both the absolute and relative amount of low-productivity inputs. Once an economic downturn eventually hits, firms shed low-productivity inputs and so the longer the expansion has lasted, the larger the absolute and percentage drop in the employment of a wide variety of inputs. To motivate our model, we document that such patterns are quite strong in US data, both in terms of employment and rental contracts. For instance, we find that five additional years of expansion lead to an additional drop of thirty percent in employment growth at the onset of a downturn.

Keywords: search, inputs, creative destruction, matching, frictions, duration, crisis, unemployment, business cycles

JEL Classification Codes: E24, E32, D92, D21, D22

*An earlier version of this paper circulated under the title: "A Forest Fire Theory of Recessions and Unemployment".

†Department of Economics, Stanford University, Stanford, California 94305-6072 USA. Jackson is also an external faculty member at the Santa Fe Institute and a fellow of CIFAR. Email: jacksonm@stanford.edu.

‡Kenneth C. Griffin Department of Economics, University of Chicago, Chicago, Illinois 60637 USA. Email: ptebaldi@uchicago.edu.

Financial support from the NSF under grant SES-1155302 as well as from Microsoft Research New England is gratefully acknowledged. We thank Bob Hall, Pat Kehoe, Christian Hellwig, Thibaut Lamadon, Pablo Kurlat, Moritz Lenel, Jochen Mierau, Luigi Pistaferri, and participants at various conferences for helpful comments.

> Capitalism, then, is by nature a form or method of economic change and not only never is but never can be stationary... This process of Creative Destruction is the essential fact about capitalism. It is what capitalism consists in and what every capitalist concern has got to live in.
>
> *Joseph Schumpeter, Capitalism, Socialism and Democracy, 1942*

1 Introduction

We provide a simple theory of cycles of growth and destruction that emerge in a production economy with search frictions, as well as empirical illustrations of these phenomena.

The intuition behind our theory and empirics is a simple observation.

Consider a firm facing an increased high demand for its products. In order to take advantage of the new profit opportunities it must expand its production. This might require renting additional space, acquiring additional machinery, borrowing money for additional inputs, and/or hiring additional employees. As there are frictions in finding any input, the opportunity cost of foregone profits pressures the firm to acquire additional inputs even if they are not the optimal ones. A firm might hire under-qualified workers, or sign a lease on an imperfect facility, or sign for a credit line at higher interest rates, than it might find if it had a long time to carefully search.

The following dynamic pattern is implied by this firm behavior. As high-demand periods persist, firms accumulate more bad matches in terms of their productive inputs (e.g. labor, space, credit, technology). So, the longer a firm has gone without a drop in its demand, the more it will tend to have acquired low-productivity inputs, in both absolute and relative terms. Thus, once the inevitable negative demand shock hits, the longer the expansion has been, the more low-productivity inputs are shed.

One of our contributions is to show that all input search models satisfying a few natural conditions exhibit such a relationship between the duration of an upswing and the shedding of inputs at the onset of a downturn. We also show that this is not just a theory, but that such patterns are strongly present in the dynamics of the use labor and machinery by US firms.

The analogy to the forest fires in our title is clear: agreeable climate conditions and the absence of sparks, droughts, or other disasters, enable the growth of weaker plants and high-fuel "underbrush" in a forest. The longer it has been since the last fire or other disaster, the greater the accumulation of fuel, and hence the hotter and more extensive the subsequent fire or decimation is when it finally appears.[1]

[1] For a recent study with extensive data, see Steel et al. (2015). Such phenomena are observed in a variety of natural settings and many species, beyond forests (for instance, see Pickett and White (1985)).

The key driving feature of this phenomenon is that firms face search frictions, thus they are not always able to find a perfect match when acquiring an input - whether it be employees, real estate, machinery, credit, or other inputs. This implies that when demand (or productivity) is high, firms may be willing to acquire inputs who are less than optimal matches rather than to wait for a subsequent better match and forego current profits.

We also show that this is true even if firms must sign long-term contracts, leases, or credit lines, as long as the current gain in profits outweigh the future discounted expected cost of the adoption of low-quality inputs.[2]

We remark that our results are not dependent on firms having to sign long-term contracts, even thought such contracts are common. The key feature that drives our results is that firms are more willing to use less-productive resources when there is higher demand. In fact, our results are proven for the case in which search frictions are constant, and the effects of our model would be amplified if we also allowed search frictions to increase as economy expanded. Such effects are well-known, as in, for example, the Beveridge Curve (Dow and Dicks-Mireaux (1958)), which notes that the available labor pool becomes tighter as the economy grows. This further increases incentives for firms to accept and retain worse-matched inputs since future search prospects worsen as the economy grows.

In particular, we examine a class of models in which firms face a basic friction in searching for inputs. We prove that Markov perfect equilibria exist (Lemma 1) and exhibit the property that the longer there is strong demand, the more poor firm-input matches build up in an economy (Proposition 4). Once an eventual crisis comes, due to a random shock in demand or aggregate productivity, the longer it has been since the last crisis, the larger the severance of inputs at the crisis onset, both in overall magnitude and in relative terms (Corollary 1). Most importantly, more inputs are shed and the downturn is larger for the same sized shock the longer the duration has been. So, the size of the downturn is driven by the length of the expansion, and one does not need a larger shock to get a larger downturn. A second set of results (Proposition 5 and Corollary 2) show that, for a given duration of subsequent positive shocks, if these were larger in magnitude, the mass of inputs being severed in response to a negative shock is also larger.

Before presenting the theoretical results, we provide novel stylized facts consistent that

[2] Long term contracts exist in most leasing and credit settings for many reasons, as the supplier has to plan investments based on the contracts and itself faces frictions. For example, landlords face substantial frictions in finding new tenants and typically prefer long-term leases over renting month-to-month, and often offer discounts for longer-term contracts. Potential hold up problems and match-specific investments also favor longer-term contracting. For instance, long-term labor contracts also appear for such reasons (see e.g. Farber, 1999), including that workers have to invest in firm-specific human capital which requires long-term contracts to avoid hold-up problems, as well as that workers are typically more risk averse than firms in facing the search frictions in the labor markets and so optimal risk sharing involves long-term contracting (see also, Azariadis (1975); Lamadon (2014)).

motivate the whole exercise. When looking at the panel of 166 US micro areas between 1969-2012, we find a positive relationship between expansion duration and the subsequent drop in employment and other inputs at a crisis onset.[3] A preview of the nonstationarity which appears in these data, is pictured in Figure 1. In this figure, we report the empirical density of the deviation in the growth of (wage and salary) jobs from its expansion average for different durations of expansion of the local economic area, once local firms experience a downturn. The figure shows that the extent to which employment drops is larger the longer the preceding expansionary period. Eight or more years correspond to almost a five times larger mean drop in job growth than two to four years of expansion.

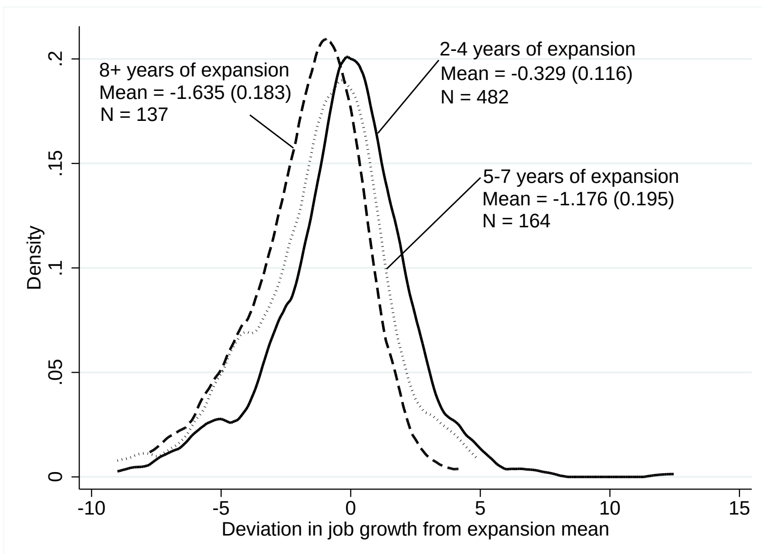


Figure 1: Empirical densities of the deviation in the growth of (wage and salary) jobs from its expansion average for different durations of expansion preceding the downturn. The data consists of 783 local crises identified in the BEA statistical areas panel from 1969-2012: we identify the onset of a downturn by a decrease in proprietors' income following two or more periods of positive growth.

Our analyses in Section 2 and Appendix 5.2 show a one-third larger drop in job growth corresponding to five additional years of expansion. Our approach is to compare areas of the same state that enter a crisis in the same year, but having had different durations of preceding expansions. For a variety specifications, and different ways of defining a 'crisis', we robustly find that longer durations correspond to significantly larger drops in employment growth.

Thus, our contribution comes in two related, but distinct, pieces. The first is documenting correlations between the length of an expansionary period and subsequent drops in employment (and rental decisions), and the second is providing a model with a simple and

[3] In this we define the onset of a local 'crisis' as a year of negative growth of proprietors' income preceded by two or more periods of positive growth, which we call 'an expansion'.

direct explanation for such relationships. The simplicity of the theoretical results fits with Occam's razor. The theoretical contribution is not to construct a complicated new model, but to show that very simple existing models actually imply the observed phenomena under some natural conditions.

1.1 Related Literature

The cyclic and destructive nature of capitalism play prominent roles in a spectrum of economic theories, from Marxian class conflict that accompanies accumulation of capital to the Schumpeterian (1939) creative destruction that accompanies innovation and growth, as noted in our opening quotation. Some prominent modern analyses of creative destruction, productivity changes, and business cycles are found in Gali and Hammour (1992), Davis and Haltiwanger (1992), Caballero and Hammour (1994, 1996), Acemoglu and Autor (2010), and Jaimovich and Siu (2012).

The key difference in our work is the emphasis on the duration of an expansion as the driver of more severe crises.

Our approach is also loosely related to the vast literature on labor search by workers and firms initiated by Diamond (1981; 1982a; 1982b), Mortensen (1982a; 1982b; 1999), and Pissarides (1984; 1985; 2000); DMP framework henceforth. This literature also includes recent works examining cyclical aspects of matchings, as in Jovanovic (1987), Barnichon (2010), Barnichon and Figura (2011), Furlanetto and Groshenny (2012), Michaillat (2012), Shimer (2005).

Although our analysis is not focused specifically on labor search, we show that the assumptions of our general model nest versions of the classical DMP framework. Here, however, we abstract away from the decisions of workers and other providers of inputs, and focus solely on firms' decisions when procuring inputs. So, we derive implications from firms' decisions and over a variety of inputs, and show new patterns both theoretically and empirically.

Lastly, counter-cyclical severance of inputs is something that has been noted in the literature. For example, an important paper by Bergin and Bernhardt (2008) provides new theory and insights into how weaker firms may stay in a market when demand is high and then exit when demand becomes low. Also related is recent work by Berger (2011). His model involves firms allowing productivity to drop in good times and then restructuring during bad times.[4]

[4]Some similar, but distinct, dynamics can be found in Koendors and Rogerson (2005), who examine the choice of production technologies with changeover costs and limitation in managerial attention. Philippon (2006) provides another micro-foundation that could also be used to drive the accumulation of bad matches during booms. In his model, managers may have incentives to over-hire for 'empire-building' motives, and this is more prevalent when there is less shareholder oversight, which correlates with booms.

Even though many models of search for labor, credit, and technology that appear in the literature fall into the models covered by our main theoretical result, this "duration-dependence" has not been noted before. A main takeaway from our results is that the "rare event" leading to a large downturn does not need to be a negative shock of exceptional magnitude, but could also be an unusually long sequence of periods with non-negative shocks which leads to a longer duration of an economic upswing. Thus, unusually large downturns do not *necessitate* unusually large shocks, but can also naturally occur after unusually long expansionary periods. The novelty in our work is not in counter-cyclical separation, but instead in providing theory and empirical evidence for the fact that the size of a downturn is positively related to the length of the preceding expansion (our "forest-fire" analogy), something that is new to the literature on both dimensions.

We begin, in Section 2, by showing that patterns of duration dependence – namely that the probability of a downturn and its severity increase with the time since the last downturn – are (strongly) present in the data. In Section 3 we exhibit a simple example of a model exhibiting this phenomenon. We then show, in Section 4, that this phenomenon is exhibited by a large class of models that share a few simple properties.

2 Stylized Facts: Expansion Duration and Drops in Employment and Rental Contracts

We begin by showing that there is a significant and quantitatively revelevant relationship between expansion duration and the shedding of inputs. Despite their large and significant nature, the empirical patterns that we show here have not been documented before: the size of employment drops when areas are hit by a negative shock to profitability positively correlates with the length of the preceding upswing.

2.1 The Data: Employment and US Micro Areas

Our analysis of employment patterns uses data from the 166 Bureau of Economic Analysis Combined Statistical Areas (BEAAs). Established by the Census Bureau, these areas define the relevant regional markets surrounding metropolitan or micropolitan statistical areas. To quote: "these economic areas represent the relevant regional markets for labor, products, and information. They are mainly determined by labor commuting patterns that delineate local labor markets and that also serve as proxies for local markets where businesses in the areas sell their products."

A BEAA contains firms that interact with each other and are subject to common profitability shocks (these could consist of demand or productivity shocks, or both); this is the level of observation at which we investigate the mechanism that we want to study. Specifically, our empirical strategies amounts to comparing different geographic areas that experienced different sequences of expansions and contractions but are otherwise similar. By looking across many areas and enough years we obtain many observations of expansions of various lengths and subsequent contractions.

We observe, for each area, a 43-year panel (1969-2012) of growth rates of several economic indicators. We focus on three key variables: number of wage and salary jobs (jobs growth), proprietors' income, and population.

2.2 Proprietors' Income and Area-Level Crises

We start by identifying years in which a given area's businesses suffered a downturn. To do so, we exploit the time-variation in the growth rate of an area's proprietors' income (yearly income of "mom and pop businesses"), a variable that we use as a proxy for the profitability of local businesses.

We define the onset of a crisis as the first year of negative growth of proprietors' income following one or more years of positive growth. We also measure the duration of this preceding expansion period.[5]

This construction returns a data set of areas and crises, where for each observation (a crisis in some area) we have the following information: area (a), year of crisis onset (y), jobs growth rate at the onset of the crisis ($e_{a,y}$), growth rate of proprietors' income at the onset of the crisis ($\pi_{a,y}$), population growth rate at the onset of the crisis ($p_{a,y}$), and duration of the expansion period preceding the crisis, in years ($t_{a,y}$). We also track the averages of jobs and proprietors' income growth rates during the expansion preceding the crisis.

Table 1 displays summary statistics for this BEAAs crises dataset, where we exclude crises for which we cannot compute the full duration (i.e., the first crisis in any given area), as well as crises with $t_{a,y} = 1$.[6]

[5] We choose proprietors' income to reduce problems of endogeneity that would arise if using personal income (containing wages and incomes of employed workers, related to employment – our dependent variable). As another alternative we could simply use employment series, and test whether the first year of negative employment growth corresponds to a larger drop the longer the period of subsequent years of positive employment growth. This creates a problem of endogenous selection into the sample. In the supplementary appendix we show that our qualitative results are robust when defining expansions and downturns directly based on employment, as well as adopting alternative definitions of crises.

[6] This latter restriction is intended to reduce the measurement inaccuracy arising from coarse, year-level observations. As reported in the supplementary appendix, our test loses power but our results are robust if we add in the one-year expansion observations, or if we use alternative definitions to identify the onset of a crisis.

Table 1: Summary statistics from the BEAAs crises dataset (growth rates are in percentages).

Variable	Notation	Mean	Std. Dev.	Min.	Max.	N
Expansion duration	$t_{a,y}$	4.897	3.611	2	26	783
Jobs growth (onset)	$e_{a,y}$	1.05	2.299	-7.43	12.58	783
Avg. expansion jobs growth	$\bar{e}_{a,y}$	1.785	1.97	-6.103	18.02	783
Propr. income growth (onset)	$\pi_{a,y}$	-7.545	7.863	-63.6	-0.03	783
Avg. expansion propr. inc. growth	$\bar{\pi}_{a,y}$	13.346	10.567	0.690	140.72	783
Population growth (onset)	$p_{a,y}$	0.966	1.285	-6.67	14.63	783
Year (onset)	y	1993.936	11	1973	2012	783
Crises per area		4.717	1.556	1	9	166
Crises per year		19.575	17.375	1	82	40

2.3 Duration of Expansions and Subsequent Employment Drops

We first show that, on average, *the longer the expansion, the larger the drop in employment at the crisis onset.* The ideal experiment we look for in the data is the comparison of areas entering a crisis in the same year (and/or state, so to control for aggregate shocks) but with different expansion durations. For example, in 2001 the Tucson-Nogales area (AZ) experienced a crisis following an 11-year expansion. In the same year, the Fresno-Madera area (CA) experienced a downturn following a shorter, 5-year expansion. We show that, *ceteris paribus*, the 2001 drop in employment growth in Tucson-Nogales should be larger, in expectation, than the one in Fresno-Madera.

Below, Proposition 4 and Corollary 1 provide a theory consistent with these relationships.

We adopt the following specification(s) with focus on coefficient γ:

$$e_{a,y} = \alpha + \gamma \times t_{a,y} \, [+\psi_y] \, [+\phi_s] \, [+\beta \times X_{a,y}] + \varepsilon_{a,y}. \tag{1}$$

$\varepsilon_{a,y}$ is a zero-mean error (for which we allow arbitrary correlation within a state-year by clustering standard errors at this level). After looking at correlations in the pooled sample, we add year (ψ_y) and (geographic) state (ϕ_s) fixed-effects,[7] while the vector $X_{a,y}$ contains additional controls: population and income growth at the crisis onset as well as averages during the expansion. The fixed effects and other controls help control for heterogeneity in the dynamics across different geographic areas, so here we try to isolate the effects of the expansion duration.

[7]Areas are also indexed by the state in which they fall within the US. We allow correlation within a geographic state.

Table 2: Estimates from equation (1): no fixed-effects.

	(1)	(2)	(3)	(4)	(5)
	Jobs growth (onset)				
Expansion duration	-0.056***	-0.077***	-0.087***	-0.092***	-0.073***
	(0.022)	(0.022)	(0.020)	(0.020)	(0.019)
Avg. expansion jobs growth		0.375***	0.126**	0.126**	0.080
		(0.055)	(0.056)	(0.057)	(0.058)
Population growth (onset)			0.733***	0.729***	0.785***
			(0.101)	(0.101)	(0.101)
Propr. income growth (onset)				0.013	0.029**
				(0.011)	(0.011)
Avg. expansion propr. inc. growth					0.036***
					(0.008)
Constant	1.324***	0.756***	0.545***	0.667***	0.246
	(0.149)	(0.154)	(0.143)	(0.166)	(0.195)
Observations	783	783	783	783	783
R-squared	0.008	0.110	0.231	0.233	0.254

OLS coefficients from equation 1. Each observation is a BEAA-year pair in the dataset described in Table 1. Standard errors in parentheses, clustered at the state-year level. *** p<0.01, ** p<0.05, * p<0.1

Results from the baseline specification are reported in column (1) of Table 2. The coefficient γ is negative and significant at any conventional level. The estimated $\hat{\gamma} = -0.056$ indicates that 5 additional years of expansion (the average in our sample is 4.9, and the median is 4) correspond to an additional 0.28 percent drop in employment growth. This magnitude increases to approximately 0.41 percent (on average) when we add controls (columns (2)-(5) of Table 2). Given the typical drop on the order of .7 percent after the onset of a downturn, five additional years of expansion drops the employment growth rate by somewhere between forty and sixty percent.

These results pool the entire variation in the data, but do not address the presence of year-specific shocks common across areas. In fact, the distribution of crises across years is very heterogeneous, suggesting the importance of aggregate US-wide effects. For example, in 2007 – the onset of the "great recession" – as many as 82 areas (out of 166) entered a crisis according to our definition.

Table 3: Estimates from equation (1) with year and (geographic) state fixed-effects.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Jobs growth (onset)				
$t_{a,y}$	-0.048***	-0.042***	-0.040***	-0.035**	-0.044**	-0.032*	-0.032*	-0.027*
	(0.018)	(0.015)	(0.015)	(0.015)	(0.019)	(0.017)	(0.017)	(0.016)
$\bar{e}_{a,y}$	0.451***	0.094	0.096	0.076	0.360***	0.109	0.108	0.088
	(0.069)	(0.080)	(0.081)	(0.079)	(0.069)	(0.078)	(0.078)	(0.077)
$p_{a,y}$		0.744***	0.744***	0.762***		0.654***	0.654***	0.675***
		(0.118)	(0.118)	(0.115)		(0.136)	(0.136)	(0.135)
$\pi_{a,y}$			-0.006	0.003			0.001	0.007
			(0.010)	(0.010)			(0.011)	(0.011)
$\bar{\pi}_{a,y}$				0.019**				0.018**
				(0.008)				(0.008)
Constant	2.220***	2.460***	2.454***	2.344***	1.343**	1.093**	1.094**	0.963*
	(0.060)	(0.062)	(0.062)	(0.080)	(0.584)	(0.524)	(0.524)	(0.508)
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
State FE	N	N	N	N	Y	Y	Y	Y
Obs.	783	783	783	783	783	783	783	783
R-squared	0.442	0.539	0.539	0.544	0.538	0.592	0.592	0.596

OLS coefficients from equation 1 including state and year fixed-effects.
Each observation is a BEAA-year pair in the dataset described in Table 1.
Standard errors in parentheses, clustered at the state-year level. *** p<0.01, ** p<0.05, * p<0.1

In our preferred specification we estimate γ including year (and state) fixed effects. This corresponds to the thought experiment of comparing similar areas entering a crisis in the same year but following expansions of different durations. The specifications in Table 3 (columns (1)-(4)) rely on this type of events in the data. The new estimates are again negative and significant, suggesting that 5 additional years of expansion correspond to an additional 0.21 percent drop in job growth. Again, given the typical drop on the order of .7 percent after the onset of a downturn, five additional years of expansion are associated with an average additional drop in the employment growth rate by thirty percent under these estimates.

Columns (4)-(8) in Table 3 present results from the same specifications where we additionally control for (geographic) state-specific fixed-effects. Despite lowering the power of our test, this addresses additional biases that might be induced by unobserved factors - such as across-state heterogeneity in regulation and type of businesses (affecting the rigidity of employment), or differences in transportation technology and geography of the state (affecting commuting patterns) - correlated with the determinants of employment dynamics within

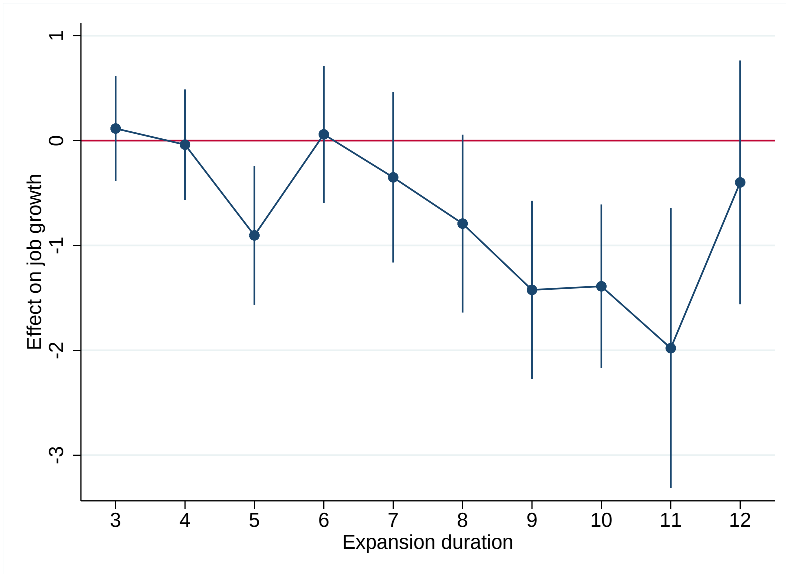


Figure 2: Estimated $\hat{\gamma}_3, ..., \hat{\gamma}_{12}$ from equation (2). BEAAs crises dataset. Standard errors are clustered at the state-year level. $t = 12$ is the 95^{th} percentile of expansion duration, coefficients $\hat{\gamma}_{13}, ..., \hat{\gamma}_{26}$ are not significant and omitted.

a state. The most conservative estimates of γ imply that five additional years of expansion correspond to an additional 0.14 percent drop in job growth.

To further explore the negative relationship between expansion duration and employment drops graphically, allowing for a nonlinear relationship between these two variables, we estimate a model with duration-specific indicators:

$$e_{a,y} = \alpha + \sum_{\tau \geq 3} \gamma_\tau \times \mathbf{1}\{t_{a,y} = \tau\} + \beta \times X_{a,y} + \varepsilon_{a,y}, \tag{2}$$

where $X_{a,y}$ includes the same controls as in column (5) of Table 2. To illustrate the negative impact of expansion duration on employment growth, in Figure 2 we plot the estimated $\hat{\gamma}_\tau$ for $\tau = 3, ..., 12$. (12 is the 95^{th} percentile of expansion duration, for larger values the estimated γ_τ is not significantly different from zero.)

2.4 Alternative test via VAR analysis

Our analysis thus far exploits only the constructed BEAAs crises dataset (we verify the robustness of our results to alternative ways of identifying crises in the supplementary appendix). In this section, we show how one could alternatively use the entire BEAAs panel to test our hypothesis, while relying less on the chosen way to define crises. Moreover, the following analysis also allows us to control more flexibly for area-specific unobservables that might affect our previous findings, not already captured by state fixed-effects.

The idea is as follows: we estimate 166 area-level vector autoregressions (VARs) using for each area the entire 43-years panel. In each VAR we allow co-integration (with 3 lags) of job growth, proprietors' income growth, personal income growth, and population growth. We can then estimate the VAR residuals for each series, in each area. These, seen as data, describe the residual variation in each area-year cell after controlling for the complex (yet assumed stationary) process underlying the evolution of the BEA variables in a given area.

The use of VAR residuals is two-fold: first, we can double check that our definition of "crisis" - relying only on the dynamics of proprietors' income - correctly identifies negative shocks along the area series of employment and personal income. Second, with a similar regression to the one used thus far, we can test whether areas with a longer expansion tend to have a lower job growth VAR residual, corresponding to the stylized fact predicted by our model.

For each area a, let $\mathbf{x}_{a,\tilde{y}} = (e_{a,\tilde{y}}, \pi_{a,\tilde{y}}, i_{a,\tilde{y}}, p_{a,\tilde{y}})$, where i denotes personal income and $\tilde{y}$ is used to denote any year in the 1969-2012 period in the full BEAA data. (We keep y to denote crises years in the BEAAs crises dataset). Also let $\boldsymbol{\epsilon}_{a,\tilde{y}}$ to be a vector of zero-mean disturbances drawn *iid* over time with contemporaneous variance-covariance matrix $\mathbb{E}\left[\boldsymbol{\epsilon}_{a,\tilde{y}}\boldsymbol{\epsilon}'_{a,\tilde{y}}\right] = \Omega_a$. We estimate the following:[8]

$$\mathbf{x}_{a,\tilde{y}} = \boldsymbol{\mu} + \boldsymbol{\Gamma}_1\mathbf{x}_{a,\tilde{y}-1} + \boldsymbol{\Gamma}_2\mathbf{x}_{a,\tilde{y}-2} + \boldsymbol{\Gamma}_3\mathbf{x}_{a,\tilde{y}-3} + \boldsymbol{\epsilon}_{a,\tilde{y}}. \tag{3}$$

We then compute, for each area, the series of residuals

$$\hat{\boldsymbol{\epsilon}}_{a,\tilde{y}} = \mathbf{x}_{a,\tilde{y}} - \left(\hat{\boldsymbol{\mu}} + \hat{\boldsymbol{\Gamma}}_1\mathbf{x}_{a,\tilde{y}-1} + \hat{\boldsymbol{\Gamma}}_2\mathbf{x}_{a,\tilde{y}-2} + \hat{\boldsymbol{\Gamma}}_3\mathbf{x}_{a,\tilde{y}-3}\right), \tag{4}$$

where we will decompose $\hat{\boldsymbol{\epsilon}}_{a,\tilde{y}} = \left(\hat{\epsilon}^e_{a,\tilde{y}}, \hat{\epsilon}^\pi_{a,\tilde{y}}, \hat{\epsilon}^i_{a,\tilde{y}}, \hat{\epsilon}^p_{a,\tilde{y}}\right)$ to keep track of the residuals for each of the four variables entering the VAR. The main variable of interest will be $\hat{\epsilon}^e_{a,\tilde{y}}$, the VAR residual of job growth in area a, year $\tilde{y}$. These residual series are thus the input for our analysis.

We start by verifying that the 783 observations in the BEAAs crises dataset correspond to area-year combinations experiencing negative shocks (not predicted by the 3-lags stationary process modeled by equation (3)). This is done in Table 4, where we compare the mean VAR residuals in the area-year pairs included in the BEAAs crises dataset to the rest of the data. This confirms that the way in which we define crises using drops of proprietors' income is consistent with the full time-variation in the BEAAs data.

[8] We use the iterated seemingly unrelated regression procedure, as implemented by the *'var'* Stata command.

Table 4: Average VAR residuals. BEAAs crises vs. rest of BEAAs data.

Average VAR residual	$\hat{\epsilon}^e_{a,\tilde{y}}$	$\hat{\epsilon}^\pi_{a,\tilde{y}}$	$\hat{\epsilon}^i_{a,\tilde{y}}$	$\hat{\epsilon}^p_{a,\tilde{y}}$
BEAAs crises dataset	-0.429	-8.379	-0.978	-0.043
	(0.058)	(0.328)	(0.078)	(0.020)
Rest of BEAAs data	0.057	1.120	0.131	0.006
		(0.024)	(0.032)	(0.008)
Difference	-0.486***	-9.499***	-1.109***	-0.049**
		(0.070)	(0.093)	(0.022)
Full dataset	5.77e-10	3.68e-09	-4.29e-09	-8.33e-10
	(0.021)	(0.140)	(0.030)	(0.007)
Observations	6640	6640	6640	6640

Note 1: For each area we can compute residuals for 40 out of 43 years.
Note 2: BEAAs crises are defined using deviations in proprietors' income.

We can then re-test the main result of proposition 4 (longer expansions correspond to deeper drops in subsequent jobs growth) by regressing the employment VAR residual on duration and year fixed-effects. This richer specification controls for aggregate year-specific shocks not captured by the area-level VARs, while using the VAR residual is assuring that our results are not driven by area-specific unobservables driving employment dynamics. The estimated equation is:

$$\hat{\epsilon}^e_{a,y} = \alpha + \gamma \times t_{a,y} + \omega_{a,y} \left[+\psi_y\right], \tag{5}$$

where we are interested in the sign and magnitude of γ and allow $\omega_{a,y}$ to be arbitrarily correlated within a given state-year cell.

Table 5 reports estimates of this equation for the full sample as well as the 1969-2005 period. Once again, this alternative test provides strong empirical evidence supporting our theoretical result. The coefficients are all significant at 5 or 10 percent level, and imply that 5 additional years of expansion correspond to an additional drop in job growth between 0.13 percent and 0.24 percent - again a large fraction of the typical drop in job growth.

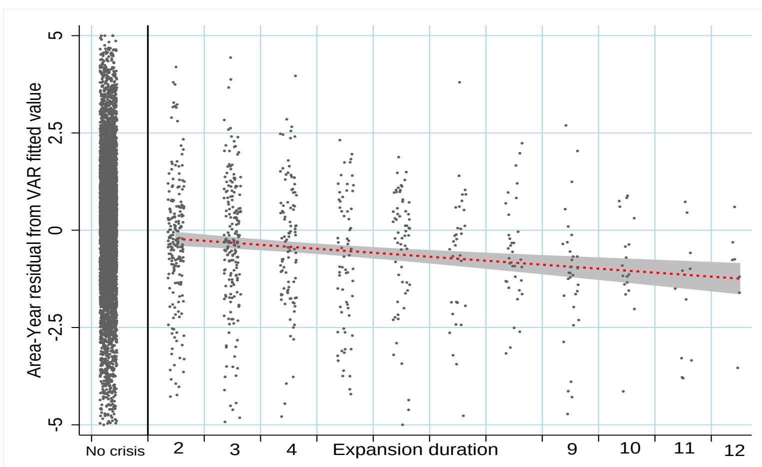
To conclude, in Figure 3 we summarize the above analysis. Each observation in the scatter plot corresponds to an area-year job growth residual ($\hat{\epsilon}^e_{a,\tilde{y}}$ on the y-axis). The large group on the right collects non-BEAAs crises, while the rest of the observations are classified according to their expansion duration. The estimates for $\hat{\gamma}$ in Table A4 are the slope of

Table 5: Estimates from equation (5).

	(1)	(2)	(3)	(4)
	Area-year VAR residual ($\hat{\epsilon}^e_{a,y}$)			
Duration of expansion	-0.034**	-0.025*	-0.048**	-0.034**
	(0.014)	(0.013)	(0.019)	(0.017)
Constant	-0.260***	-0.307***	-0.187	-0.252**
	(0.099)	(0.093)	(0.116)	(0.102)
Sample	1969-2012	1969-2012	1969-2005	1969-2005
Year FE	N	Y	N	Y
Observations	783	783	633	633
R-squared	0.006	0.310	0.009	0.337

Robust standard errors in parentheses, clustered at the state-year level.
*** p<0.01, ** p<0.05, * p<0.1

Figure 3: Graphical representation of the test based on area-level VARs. Each observation represent an area-year job growth residual ($\hat{\epsilon}^e_{a,\tilde{y}}$, y-axis). We separate non-crisis observations (first class on the left) from BEAAs crises. Our test relies on the slope of the dotted line: impact of expansion duration on drop in job growth conditional on being at a crisis onset.



the dotted line, representing the effect of an additional year of expansion on the drop in employment, conditional on the area being in the first year of a crisis.

2.5 Similar Patterns in Rental Decisions

Our empirical analysis so far has focused on the relationship between changes in employment and duration of expansions.

The logic behind this applies to any factor of production which has search frictions and match quality. This is true of rental space, equipment, and any technology used in production: they require some search to acquire and can have qualities of match. For example, a rental space has size, location, and other qualities, that may match well or poorly for a given firm.

This is also verified empirically, as we now show: there is a significantly positive relationship between the duration of an expansion and the subsequent drop in rental commitments at the onset of a crisis.

To explore this empirically, we use the COMPUSTAT panel of firm-year-level financial data from 1950-2012, along with BEAA industry-year-level data to identify crises.

We repeat the same procedure we conducted with proprietor's income in Section 2 with the series of chain-type price indexes for value added. The onset of an industry-level crisis is identified by a drop in the industry-specific price index for value added following more than one year of positive growth of the same variable. We similarly compute the duration of the expansion period preceding a crisis, the key explanatory variable of our analysis.

We merge these variables (varying at the year-industry level) with the COMPUSTAT panel, creating an indicator taking value 1 if the firm operates in a sector (3-digit NAICS classification) entering a crisis in a given year, and recording the duration of the expansion before the crisis occurs.

To test for the predictions of our model, we examine the relationship between duration of the industry-expansion and firms' decisions about two distinct classes of inputs: employment (replicating our previous results in this different context) and operating rental commitments of the firm at the end of the fiscal year. In particular, we consider the value of all the contracts – typically including rental of office space and capital equipment – with duration between 1 to 5 years, transforming this nominal variable in real terms using the nation-wide GDP deflator.

For every firm j and year y, we compute the year-to-year percentage change in employment and year-to-year percentage change in real rental commitments – $e_{j,y}$ and $r_{j,y}$, respectively. Using $y_{j,y}$ to represent either of these variables, the regression equation used to test our hypothesis is:

$$y_{j,y} = \alpha^0 + \mathbf{1}(crisis_{j,y} = 1) \times (\alpha^1 + \gamma \times t_{j,y}) + \psi_y + \varepsilon_{j,y} \; [+\phi_{NAICS}] + [\beta \times \Delta EBIT_{j,y-1}]. \tag{6}$$

The variable $crisis_{j,y}$ is the crisis-dummy constructed from the BEA price index data: it takes value 1 if the industry where firm j operates experiences a drop in price index for added value computation in year y. $crisis_{j,y} = 1$, $t_{j,y}$ measures the duration of the preceding expansion in the same industry. The coefficient γ is again our main focus, where a significantly negative coefficient indicates that a negative relationship between expansion duration

Table 6: Estimates from equation (6).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dep. Var.	$\Delta Rents$	$\Delta Rents$	$\Delta Rents$	$\Delta Rents$	$\Delta Empl.$	$\Delta Empl.$	$\Delta Empl.$	$\Delta Empl.$
$t_{j,y}$	-0.00968***	-0.00492*	-0.0107***	-0.00650**	-0.00128*	-0.00163**	-0.00120	-0.00146*
	(0.00259)	(0.00269)	(0.00257)	(0.00268)	(0.000762)	(0.000777)	(0.000758)	(0.000775)
$\Delta EBIT$	N	N	Y	Y	N	N	Y	Y
Year Fe	N	Y	N	Y	N	Y	N	Y
Industry Fe	N	Y	N	Y	N	Y	N	Y
N	108149	108149	102524	102524	162019	162019	151298	151298
R^2	0.000197	0.0110	0.000285	0.0102	0.0000410	0.0246	0.0000962	0.0235

and change in either employment or real-valued rental commitments is observed in the data. The variable ψ_y denotes a year-specific fixed-effect. Industry-specific (ϕ_{NAICS}) and lagged percentage change in real-valued EBIT (Earnings Before Interest and Tax) are included in some specifications, with the latter controlling for firm-specific changes in profitability not related to the industry-level price shock.

Table 6 reports estimates of γ for various specifications, with the dependent variable corresponding to change in real-valued rental commitments (columns 1-4) or employment (columns 5-8), respectively.

We find a negative and significant effect (across a variety of specifications) of the duration of the industry expansionary period on these adjustments at the onset of a crisis.

3 A Simplified Model

We now build a simplified model that shows an easy way to generate the sorts of nonstationarities, and link between the duration of an expansion and the size of a downturn, that we just showed exists in the data.

In the simplified model the match between an input and a producer (or firm) is either "good" or "bad" and the productivity (or demand) state is also bivariate, either "high" or "low". In Section 4, we present results for any (compact) set of match types, showing that our results extend to a wide set of possible models.

3.1 Timing and a Firm's Input Search

A firm looks to acquire a (marginal) unit of input.[9] Time comes in discrete periods $t \in \{1, 2, \dots\}$. If the firm does not currently have the input, then it gets to observe the random

[9]The extension to many inputs involves complications of specifying details of a production function. To highlight the dynamics, we work with the simplest case and just focus on one unit of input on the margin.

arrival of a potential match.[10] The firm decides either to acquire the input, or to wait and search again. In this case, the firm earns 0 (marginal) profit for that period and awaits a new match in the next period.

Let the quality of the available input be either "good" or "bad," denoted $m_t \in \{G, B\}$. The quality is good with probability v, bad with probability $(1 - v)$. Once the input is acquired the quality stays the same for all periods in which the input is utilized. If the input is severed, then the quality of the potential match in the next period is independent of the quality of past matches (both accepted and rejected).

There is an aggregate productivity (or demand) state, either high or low, denoted $s_t \in \{H, L\}$. For the one-firm case, we take this to be independently and identically distributed across periods, being H with probability p and L with probability $(1 - p)$. These states can be basically interpreted as anything that might vary with time and affects how input quality translates into profits.

In particular, input quality affects the profits of the firm π_{sm}, along with the aggregate productivity state:

- profits from good inputs are positive regardless of the state, $\pi_{HG} > \pi_{LG} > 0$;
- profits from bad inputs are only positive in the high state, $\pi_{HB} > 0 > \pi_{LB}$; and
- good inputs are more profitable than bad ones in each state s, $\pi_{sG} > \pi_{sB}$.

These are weak requirements: high states are more profitable than low states, good matches are profitable in all states while profits from bad matches are only positive in the high state, and good matches are more profitable than bad ones. These requirements are essentially tautological, we could simply use these requirements to define what it means to be a good versus a bad match.

Note that we abstract away from modeling input prices (e.g. wages, interest rates, prices of intermediate goods) explicitly, and simply roll them into profits directly. Effectively, any model for which net-of-price profits from good inputs exceeds those of bad inputs, and bad inputs are profitable only in the high state, satisfies our conditions regardless of how prices are set.

The firm discounts the future according to a discount factor $\delta < 1$, and maximizes its expected stream of discounted profits. The firm perfectly observes the quality of the input and the productivity state (or at least some signals correlated with these[11]) when deciding

[10] A natural extension is that, as typical in labor-search models, it takes some (random) time for the firm to even observe the potential match. We deal with that case in the Appendix 5.4.

[11] The model works just as well if everything is done in terms of expectations, with noise in the realized qualities and states. Firms would still be willing to take on lower expected productivity inputs in states that are expected to be more profitable.

on whether to acquire the available input. Inputs are severed randomly with a probability $q \in [0,1]$ at the beginning of any period (excluding the period in which they are first acquired). This can be thought as of workers quitting, machinery breaking, or credit lines expiring or disappearing.

So, the ordering within a period is as follows:

- At the beginning of the period, existing firm-input matches are dissolved with probability q.
- Next, the state is realized and observed by the firm.
- Then, if the firm has no input, it gets a new match (whose quality it observes) and it makes a procurement decision.
- Finally, profits are realized.

3.2 A Firm's Optimal Decision

All that is necessary for our results is that once the firm acquires the input (e.g. hires a worker or buys or leases some machinery), the firm is willing to keep this input, regardless of quality, as long as demand stays high. In this section, we consider the more complicated case in which the firm must keep the input indefinitely (except for the exogenous separation). By showing that the firm is still willing to acquire bad inputs even under such circumstances, the results hold *a fortiori* if the firm can sever matches when the demand state is low.

Consider a period t in which the firm is searching for an input, a match m_t is possible and the realization of the state is s_t. In an optimal strategy, the firm will clearly acquire a good input regardless of the state. It is also possible that, in an optimal strategy, the firm would acquire a bad input. In this simple version of the model, this can only happen in a high state. Therefore we consider two possible strategies: $\sigma \in \{\sigma_g, \sigma_{Hb}\}$ where σ_g means that a firm only acquires good inputs regardless of the state, and σ_{Hb} means that a firm acquires good inputs in any state but also bad inputs when the state is high.

Let $E_s(\pi_{sG})$ and $E_s(\pi_{sB})$ denote the expected profits conditional on having a good/bad input, respectively. Denote by $V(\sigma)$ the expected stream of discounted profits, beginning at the start of a period without an input, and following a strategy σ. Straightforward calculations (details appear in Appendix 5.4) show that

$$V\left(\sigma_g\right) = \frac{vE_s\left(\pi_{sG}\right)}{\left(1-\delta\right)\left(1-\delta\left(1-q\right)\left(1-v\right)\right)} \tag{7}$$

and

$$V\left(\sigma_{Hb}\right) = \frac{vE_s\left(\pi_{sG}\right) + p\left(1-v\right)\left(\delta(1-q)E_s\left(\pi_{sB}\right) + \left(1-\delta\left(1-q\right)\right)\pi_{HB}\right)}{\left(1-\delta\right)\left(1-\delta\left(1-q\right)\left(1-\left(v+p\left(1-v\right)\right)\right)\right)}. \tag{8}$$

The following proposition then follows from the expressions for the value functions above.

Proposition 1 *Acquiring bad inputs in high states is an optimal strategy if and only if*

$$\frac{vE_s\left(\pi_{sG}\right)}{1-\delta\left(1-q\right)\left(1-v\right)} \leq \frac{\pi_{HB}}{\delta(1-q)} - \left(\pi_{HB} - E_s\left(\pi_{sB}\right)\right). \tag{9}$$

The comparative statics are intuitive. Acquiring bad inputs in a good state is more attractive (i.e., the left hand side of (9) decreases and or the right hand side increases) as: (i) good matches become less likely (v decreases), (ii) the firm becomes less patient (δ decreases), (iii) matches are dissolved with higher probability (q increases), (iv) profits from good inputs (in either state) decrease (π_{sG} for either s decreases), and (v) profits from bad inputs (in either state) increase (π_{sB} for either s increases).

3.2.1 Firm-Level Crises and Input Severance

Let a firm "crisis" be a situation in which the state is low and the firm has a bad match in place: $s_t = L, m_t = B$.

A firm crisis would be a situation in which a firm's input match is leading to negative profits. In such situations a firm will shed its input if it can, or as soon as it can, and may even do so via bankruptcy.

Proposition 2 *Suppose that (9) holds, so that a firm is willing to acquire bad inputs in the high state. Consider starting at some date t in the high state H and a good match $m_t = G$ and then consider τ subsequent periods of high states. The probability that $m_{t+\tau} = B$ increases with τ, and so the probability of a firm crisis occurring in period $t+\tau+1$ also increases in τ.*

Proposition 2 shows that firms are increasingly likely to have a crisis the longer the expansionary period before a low-state shock. The intuition is that the firm is more likely to have replaced the input the longer the duration of expansion, and hence the more likely the firm is to have acquired a bad input.[12]

[12] The probability of having a bad match in place asymptotes upward towards the steady-state probability that a firm would have a bad match in place if the state were always high.

3.3 Interactions Between Profits of Multiple Firms

We now examine an overall economy, considering multiple firms making decisions. The interest in this case comes from the fact that the interaction between firms may affect their profitability.

We work with a unit measure of atomless firms so that a firm does not anticipate its individual decision influencing economy-wide dynamics.

At any point in time, each firm has one of three match types: G, B, U where U means that they are currently unmatched. The relevant description of matches in the economy is g_t, b_t, u_t, which is the fraction of firms that have each type of match at the beginning of period t, and where $u_t = 1 - g_t - b_t$. Thus, it is sufficient to keep track of g_t, b_t.

This version of the model is the same as the single-firm case except for the following. Firms interact with each other. In a high state, all firms keep their inputs. In a low state, some firms with bad inputs can be forced to shut down or shed their inputs. We refer to this as a "firm-level crisis", and treat this as if the firm shed its input and goes back to its initial unmatched state, or is equivalently replaced by a new firm.

Note that this tempers the results as it makes firms even more reluctant to acquire bad inputs, and again so the results hold a fortiori in the case where the firm is just forced to sever inputs yet survives.

In particular, we consider settings in which a firm is more likely to be forced to layoff its worker as the fraction of bad matches $(b_t/(g_t + b_t))$ increases. So, as the overall productivity in the economy decreases, individual firms are more susceptible to crises. The probability that a given firm with a bad match has a crisis in a low state is given by $\phi\left(\frac{b_t}{g_t+b_t}\right)$, where ϕ is a non-decreasing function.

A simple example of this is one in which firms are paired together, for instance in some business venture, or have a vertical relationship in which one supplies the other. If two firms with bad input matches are paired and the state is L, then they both suffer. For instance, low demand causes lower sales for a retailer, which then affects a supplier to that retailer, and if both have bad inputs, they each suffer to the point of having to shut down and sever their physical and labor inputs. For simplicity, we assume the replacement of firms so that the mass of active firms is constant and equal to one.[13]

[13]This interaction between firms corresponds as captured in ϕ to a situation in which firms are complements. This comes both from direct interactions as well as interactions with the whole economy: firms have crises with a higher rate as the overall health of firms in the economy is weaker. The alternative situation would be one where weak firms benefit from others' weaknesses. Although this may seem intuitive, fewer firms tend to be in direct competition than in some other relationship. In fact, as pointed out by Acemoglu and Autor (2010) the complements case seems more relevant. For example, as they point out, Ford Motor Company lobbied the U.S. Congress to help bail out General Motors, since they share many parts suppliers. If G.M. had failed that would have hurt the suppliers and the loss of suppliers would have hurt Ford more than they would gained from having lost a competitor in the car market. Even so, it is not necessary to

In this setting, the firms' procurement decisions are more complicated since they care about their interactions with other firms and so condition their decisions on the distribution of matches in the economy g_t, b_t, in addition to the demand or potential-profit state. Firms can still be willing to acquire bad matches in high states, regardless of the distribution of matches.

For the following result we analyze a situation in which firms acquire bad matches in a high state, regardless of the current distribution of the matches in the economy, but do not acquire bad matches in a low L state. This holds for a range of parameter values. Also, we assume that the initial match distributions, b_0 and g_0, are below their maximum steady-state levels.[14,15]

Proposition **3** *Consider an economy emerging from a low demand state at time t with a ratio of bad matches that is not excessively high ($\frac{b_t}{g_t} < \frac{1-v}{v}$), and τ subsequent periods of the high state. Longer expansionary periods (larger τs) lead to:*

(a) *higher overall fraction of firms with inputs (higher $g_{t+\tau} + b_{t+\tau}$),*

(b) *more bad matches, both in number and as a fraction of matches (higher b_t and $b_t/(g_t + b_t)$), and*

(c) *more firms, both in number and as a fraction of firms, sever their input in the next low state.*

Proposition 3 shows that the dynamics that occur on a single firm basis extend to an economy. In fact, higher factions and numbers of firms sever inputs the longer the expansionary period, keeping fixed the size of the exogenous shock (here binary). This is the connection to the forest fire analogy: poorer input-to-firm matches accumulate as an expansion endures increasing the susceptibility of firms to crises and the size of the downturn when it eventually occurs. The event driving a large drop in inputs (e.g. employment) need not be an unusually large shock but the large drop is a natural consequence of many periods of accumulated lower-quality matches adding more fuel for a larger fire when a spark eventually hits.

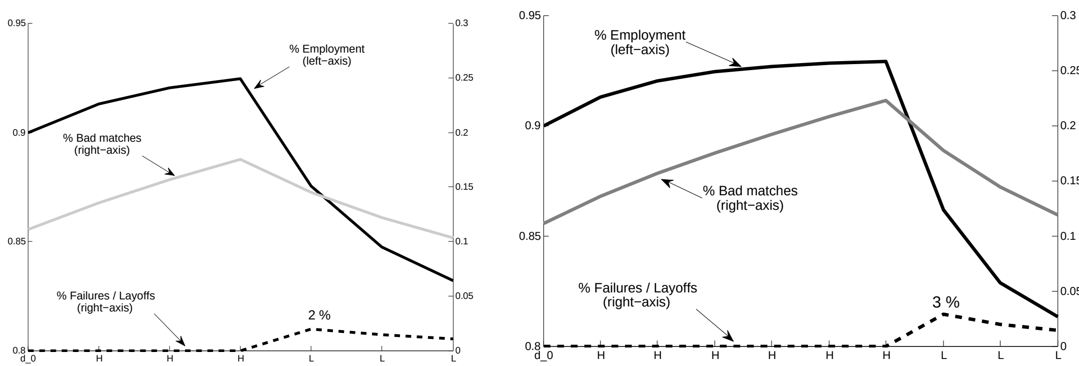
A simple illustration of these dynamics appear in Figure 4 for a particular example of parameters ($q = 0.05$, $v = 0.5$, $\alpha = 0.4$, with$b_0 = 0.10$ and $g_0 = 0.80$, and $\phi = b/(b+g)$).

---

have firms' layoffs be positively correlated for our results to hold – it is only necessary that they are not too negatively correlated. Firms will tend to want to hire bad matches in high states, and all that is necessary is that this tendency not be overly counter-acted.

[14] The economy has two steady-state levels, one that would be reached if the state stayed high forever (this is the maximum for b), and another if the state stayed low forever (the maximum for g).

[15] The proofs in Appendix 5.4 are written for the more general case with random arrival of possible matches at rate $\alpha \in (q, 1]$.



(a) Three-period expansion (b) Six-period expansion

Figure 4: Illustration of Proposition 3: In the two panels the starting point of the economy is the same with equal levels of the input (in this case referred to as "employment" on the y-axes) and equal state s=d_0 (x-axis). In panel (a) the economy has three periods of s=H before s=L occurs, while in in panel (b) the economy has six periods of s=H before s=L occurs. The share and number of firms shedding the input is higher in panel (b).

In addition to what happens at the onset of a crisis, we can also deduce some dynamics within a crisis (consecutive periods with low states). In particular, in Proposition 6 in Appendix 5.4, we show that the longer a crisis lasts, the lower the number and the fraction of firms that experience a crisis and drop inputs per period and the higher both the fraction and mass of good matches in the economy.

4 A General Class of Models

We generalize the above model to show that all models satisfying some basic properties exhibit similar patterns.

4.1 The Setting

An economy consists of a unit mass of non-atomic firms.

The set of 'states' that affect profitability of the firms is a compact set $S \subset \mathbb{R}^{\ell}$. We write $s = (s_1, \ldots, s_\ell) \geq s' = (s'_1, \ldots, s'_\ell)$ if each of the dimensions of the state $s \in S$ are at least as large as those of $s' \in S$.

The state includes things like demand, productivity, costs, etc.

We track the acquisition of inputs to firms via a "matching" process, that captures the frictions in search.

The set of possible types of the input that a firm may be matched with lies in a compact set $M \subset \mathbb{R}$.

The matches in the economy at time t is described by (u_t, μ_t), where $u_t \in [0,1]$ is the fraction of unmatched firms and μ_t a probability distribution on M, $\mu_t \in \mathcal{P}(M)$, which describes the relative frequencies of types of matches among firms that currently have inputs.[16]

The timing of period t is the following:

1. State realization: A state s_t is drawn from a distribution $\eta \in \mathcal{P}(S)$ and publicly observed. To emphasize that the nonstationarity of crises in our model is not driven by underlying nonstationarities, we take the draw of s to be i.i.d. over time.[17]

2. Crises/Layoffs: A firm experiences a crisis and drops its input (severs its match) with a probability ϕ that depends on the quality of the input, the state, and the distribution of matches in the economy: $\phi(\mu_{t-1}, s_t, m)$. In particular, $\phi : \mathcal{P}(M) \times S \times M \to [0,1]$ is continuous and non-increasing in μ, s, and m.

3. Exogenous separation: Remaining matches are exogenously separated with probability $q \in [0,1]$. This corresponds to the lowest, base rate of separation that occurs regardless of the state and matches of inputs to firms.

4. Procurement: Active unmatched firms find a new input with a type that is randomly drawn from a (non-atomic) distribution F.

5. Profits are recorded: An active firm matched to an input of type m input earns profits $\pi(m, s)$, where $\pi : M \times S \to \mathbb{R}$ is bounded and increasing in both arguments. An active unmatched firm has zero profits.

6. Entry: The measure of firms that have had crises close and are replaced by an equal mass of new firms that become active and unmatched.

The new distribution of matches u_t, μ_t is publicly observed.

Firms maximize the discounted sum of expected profits over time with a discount factor $\delta \in (0,1)$.

We have been deliberately vague in micro-founding the setting here, as this admits many models as special cases. The restrictions, for instance, that profits increase in both the state and match quality hold in a multitude of models (with imperfect competition).

[16] In discussing convergence and domination among distributions, we endow $\mathcal{P}(M)$ with the weak*-topology and the first-order stochastic dominance partial order.

[17] Our analysis extends to settings in which the state evolves as a monotone dynamic process, such that the distribution of the state has a stochastic dominance (or more generally, positive association) relationship as a function of the past state and other attributes of the setting. Of course, one could instead directly derive the results of deeper crises given longer expansions by assuming it in terms of the distribution on states, but our goal here is to emphasize the natural economic mechanism that generates such dynamics.

4.1.1 An Example: the Mortensen-Pissarides Model of Labor Search

For example, a model that fits the above as a special case is the classic Mortensen and Pissarides (1994) (henceforth, M-P) model and which has been used extensively (e.g., see Shimer (2005)).[18] In translating notation, the M-P variables p, ε translate to our s, m with our discount factor $\delta = 1/(1+r)$, where r is their interest rate r.[19]

M-P profits are given by a linear function of s and m:

$$\pi(m, s) = (1-\beta)(s+m),$$

where β is the M-P bargaining term (and wages to the worker are the remaining part of the surplus $\beta(s+m)$).

In M-P, $q = 0$, so jobs only are terminated due to low productivity. In M-P layoffs occur with certainty if profits fall below a given level and not otherwise. In particular, the layoff function from their model would be

$$\phi(\mu_{t-1}, s_t, m) = 1 \text{ if } s_t + m < X \text{ and } \phi(\mu_{t-1}, s_t, m) = 0 \text{ otherwise,}$$

where X is an equilibrium expression defined on the right hand side of (10) in M-P .[20]

The added feature in the M-P model that is not included in our specification is the presence of a "matching function" allowing the probability that a firm with a vacancy meets a worker to depend on u_t in an increasing manner. In particular, in M-P the probability of meeting a worker increases with the number of unemployed workers relative to vacancies.[21] This is easy to add to the model, having the arrival rate of new matches to firms with vacancies following a function that increases in the (relative) number of unemployed workers. This would amplify our results, as it increases the incentives for firms to hire workers as the duration of an expansionary period increases, since matches become scarcer and so the waiting time for the next match goes up and a current match becomes more attractive.

[18] We discuss a discrete time version.

[19] M-P normalize their ε to have unit variance, and then consider the variable $\sigma\varepsilon$ as the job quality, which is what translates into our m. To simplify notation, we drop their normalization and treat ε as the job-specific quality, which is equivalent to our match-quality.

[20] Strictly speaking, their ϕ violates our continuity assumption. However, continuity of ϕ is just used for existence in our more general model, but can be relaxed in special cases for which equilibria exist.

[21] This refers to the matching function $m(u, v)$ function in their model, which is a meeting probability depending on the number of unemployed workers and vacancies, which is generally normalized to depend on v/u, which would translate to function of the u_t in our model, as we do not track the number of unemployed workers. They have an added feature of job creation, that we leave out for simplicity in our model.

4.2 Monotonicity and Stationary Strategies

A history of the economy through period t is an array

$$h_t = (u_{t-k}, \mu_{t-k}, s_{t-k})_{k=0}^{t}.$$

Because firms are non-atomic, the decision of a single firm does not affect the transition of the aggregate state of matches $u_{t-1}, \mu_{t-1} \rightarrow u_t, \mu_t$ or the state $s_t \rightarrow s_{t+1}$. Thus, a firm's future expected payoffs are increasing in its match-type m: for any (end of) period in which a firm has a match, the firm's profit, $\pi(m, s)$, is increasing in m and its probability of transitioning to a crisis is non-increasing in m given any u, μ and s.

Maximization of expected discounted streams of profits implies that in any given period t, and for any history h_t, a firm has a threshold $\widehat{m}_t(h_t) \in M$ such that a possible match is accepted if it exceeds $\widehat{m}_t(h_t)$ and not if it is below. The firm may mix at atoms, but regardless, this allows us to represent a strategy of a firm in terms of its probability of hiring during a given period.

So, a matching strategy $\sigma(h_{t-1}, s_t) \in [0, 1]$ indicates the probability that a firm accepts a potential match conditional on the history through the previous history and the current state.

In this setting, an equilibrium exists in stationary strategies, allowing decisions in period t to depend only on the current-period state $(u_{t-1}, \mu_{t-1}, s_t)$.

4.3 Equilibrium

When the other firms in the economy adopt a given matching strategy, σ, a firm (being atomless) then faces the transition $\mathbf{T}^{\sigma}$ describing the evolution of the state $(u_{t-1}, \mu_{t-1}, s_t)$ and chooses its own matching strategy optimally.[22]

An equilibrium is a matching function σ^* such that:[23]

1. σ^* is optimal (among all strategies, including nonstationary ones) for all histories and states for a firm taking $\mathbf{T}^{\sigma^*}$ as given

2. $\mathbf{T}^{\sigma^*}$ is generated by σ^* (and the other elements of the model).

[22]The function $\mathbf{T}^{\sigma}$ also depends on the crisis function ϕ, the distributions F and η, and the parameter q as detailed in Appendix 5.3. We omit the full notation to keep the presentation uncluttered.

[23]This definition of equilibrium allows for a continuum of agents and circumvents some measurability assumptions by allowing the measures to aggregate the individual actions in an anonymous manner given the atomless nature of the setting. This is a standard technique, which dates to Hildenbrand (1974) and has been extended and used in a variety of settings, including Bodoh-Creed (2012) and Adlakha et al. (2011).

In this setting an equilibrium exists, as shown in Appendix 5.3.[24]

LEMMA **1** *For any model satisfying 1-6 from Section 4.1, there exists an equilibrium σ^*, and any equilibrium is such that the matching strategy $\sigma^*(u_{t-1}, \mu_{t-1}, s_t)$ is non-decreasing in the state s_t.*

The assumptions are weak enough so that there may exist multiple equilibria, but our results apply to directly to any equilibria that satisfy minimal conditions discussed below.

4.4 Dynamics

We now analyze the dynamics of matches in equilibrium.

One condition that is needed for the more general model to exhibit dynamics similar to those of the simple model, is that the function ϕ be such that the match state of the economy not feedback so much as to reverse decisions by firms. So, with an equilibrium strategy σ^* such that, absent ϕ, the distribution of matches gets worse (in the first-order stochastic dominance sense) going from t to $t+1$, the "cleansing" effect of ϕ cannot be so large as to reverse the ordering.

In particular, a condition on the primitive function, ϕ, is needed and not on firms' behavior. Typically in equilibrium, firms' behaviors would not be so conservative (e.g., lowering the probability of acquiring an input) to reverse the ordering of the distribution of matches when transitioning between two periods. A (non-atomic) firm would deviate from such a strategy and hire worse matches, if the rest of the economy were improving. Nonetheless, the function ϕ, could have this effect, since as matches in the economy get progressively worse a larger number of those matches could get destroyed.

For this reason, in what follows we work with the following "no reversals" condition.

A state u, μ *dominates* another state u', μ' if $u \geq u'$ and μ (weakly) first order stochastic dominates μ'. Domination implies that there are both absolutely and relatively more "bad" matches under u', μ', since the distribution is worse and there is a higher number of firms with a match. We say that there is *strict dominance* if both inequalities are strict.[25]

An economy and a strategy σ satisfy *no reversals* if u_{t-1}, μ_{t-1} dominates u'_{t-1}, μ'_{t-1} implies that u_t, μ_t dominates u'_t, μ'_t for any s_t, where $u_t, \mu_t = \mathbf{T}^\sigma(u_{t-1}, \mu_{t-1}, s_t)$ and $u'_t, \mu'_t = \mathbf{T}^\sigma(u'_{t-1}, \mu'_{t-1}, s_t)$.

We state this as a condition on T rather than ϕ, since this is weaker than imposing sufficient conditions on ϕ alone, since ϕ can permit some reversals, but how much depends on

[24] Bergin and Bernhardt (1992) prove existence of equilibria in a class of related dynamic models, but their results would not apply to our model without stronger assumptions. Thus, we provide a direct proof of existence tailored to our model.

[25] Strict first order stochastic dominance here means that the two distributions differ and satisfy weak dominance.

how firms' hiring behavior overcomes the reversals. A variety of conditions on ϕ are sufficient for this to hold, the simplest one being that ϕ does not depend on μ_{t-1}. More generally, no reversals requires that, if μ_{t-1} dominates μ'_{t-1}, the difference $\phi\left(\mu'_{t-1}, s_t, m\right) - \phi\left(\mu_{t-1}, s_t, m\right)$ is bounded by some number that depends on s_t, m, F, η, q, π, and δ. [26]

Our next results show our main point that the longer the expansionary period, the worse the expected match distribution, and state, and the more firms that are expected to experience crises and separate from their matches.

Consider an initial state s_1, and let

$$G_t\left(s_1\right) = \left\{\left(s_1, ..., s_t\right) \in S^t : s_{\tau+1} \geq s_\tau,\ \tau = 1, ..., t-1\right\},$$

be all the possible t-period expansions, so that state increases for those t periods.

Given some threshold strategy, σ, and some initial aggregate match state, u_1, μ_1, let

$$EM_t^\sigma\left(u_1, \mu_1, s_1,\right) = E\left[M_t^\sigma\left(u_1, \mu_1, \mathbf{s}\right) | \mathbf{s} \in G_t\left(s_1\right)\right]$$

be the expected match distribution in the economy in period $t+1$ conditional on $\left(u_1, \mu_1, G_t\left(s_1\right)\right)$.

Proposition **4** *Consider any model satisfying 1-6 from Section 4.1, and an equilibrium σ^* that satisfy no reversals. For any initial aggregate match and states, u_1, μ_1, s_1, the expected match distribution conditional on t periods of expansion, $EM_t^{\sigma^*}\left(u_1, \mu_1, s_1\right)$, is weakly decreasing in the number of periods, t, in our dominance sense.*

Corollary **1** *Consider any model satisfying 1-6 from Section 4.1, and an equilibrium σ^* that satisfy no reversals. For any initial aggregate match and states, u_1, μ_1, s_1, the longer the expansion (the greater the number of increasing states t), the larger the fraction of firms that have a crisis for any given state s_{t+1}.*

Corollary 1 embodies a main theoretical prediction of our class of models, extending the results of Proposition 3 to this much more general setting. Thus, just as in the model analyzed in Section 3, the duration of an expansionary period decreases the overall quality of matches and increases the fraction of firms going bankrupt for a given drop in the state s.

In the richer environment of the full class of models, it is also true that the *size* of the expansion (e.g., in terms of increases in demand) has a similar effect, since equilibrium matching $\sigma^*\left(u, \mu, s\right)$ is non-decreasing in s. This is a standard feature of search models, but was not embodied in the simple model with the binary shock.

[26] This upper bound on the separation of matches as a function of the state of the economy could be derived analytically or numerically by making functional form assumptions on the model primitives.

Consider t periods, with corresponding states $\mathbf{s} = (s_1, ..., s_t)$ and some starting aggregate match distribution u_1, μ_1. Given some threshold strategy σ, let $M_t^\sigma(u_1, \mu_1, \mathbf{s})$ be the match distribution u_t, μ_t in the economy in period t conditional on the starting match distribution and the sequence of states $(u_1, \mu_1, \mathbf{s})$.

PROPOSITION **5** *Consider any model satisfying 1-6 from Section 4.1, and an equilibrium σ^* that satisfy no reversals. For any initial distribution u_1, μ_1, if $\mathbf{s}' = (s_1', ..., s_t') < \mathbf{s} = (s_1, ..., s_t)$, then $M_t^{\sigma^*}(u_1, \mu_1, \mathbf{s}')$ dominates $M_t^{\sigma^*}(u_1, \mu_1, \mathbf{s})$.*

The following corollary then shows that the fraction of firms going bankrupt is increasing in the history of states, so that a higher sequence of states has led to worse matches and hence more separation of existing matches for any given circumstance in the next period.

COROLLARY **2** *Consider any model satisfying 1-6 from Section 4.1, and an equilibrium σ^* that satisfy no reversals. For any initial distribution u_1, μ_1, and a sequence of t periods with states $\mathbf{s}$, the fraction of firms having a crisis for a given state s_{t+1} in the next period is increasing in $\mathbf{s}$.*

We remark that the results here actually imply a wide set of effects that all lead to greater downturns following longer expansions. We have emphasized 'crises' - but more generally the results are that firms have greater fractions of bad matches in their inputs. This not only makes them more likely to fail, but also makes them more likely to release their bad matches, as well as making them less profitable and hence less likely to match to new inputs (of any quality). Thus, we can see greater effects both in increasing separation and decreasing matching rates (see also, Section 5.1).

4.5 Discussion of the Relation Between the Data and the Theory

Our empirical results show that US firms exhibit patterns that are significant in the ways predicted by our theoretical results. Of course, we cannot be sure of causation from such observational data; but the very specific patterns predicted by the theory are seen in the data. These patterns are also robust to measurement: alternative investigations of consistency of the data with the implications of our model are presented in Appendices 5.2 and 6.

There are also other potential explanations for such patterns in the data, and so the point of the data is not to *prove* that the theory is the (only) explanation. Things like empire-building (e.g., Philippon (2006)), which builds from looser shareholder attention in booms, or bank investment decisions (e.g., Brunnermeier and Sannikov (2014)) that are based on VAR models that will vary positively with an expansion; could produce a similar pattern in some circumstances. It is possible that several forces are at work. Nonetheless, the theory and data presented here are both new and of independent interest. Detailed empirical tests of the various mechanisms *causing* the patterns in the data provides a rich and challenging agenda for further research.

5 Concluding Discussion

Looking at US employment series and rental commitment, we have found a novel fact: the drop in jobs and rentals during an economic downturn is significantly correlated with the length of the preceding expansionary period.

We have shown that in a simple but general class of models, firms facing search frictions in acquiring productive inputs are willing to accept relatively lower-productivity matches in periods with high demand or profit potential. These lower-quality matches build up during an expansion, then leading to larger separation (severance of existing matches, e.g. labor contracts, procurement contracts, or credit lines) as an economy eventually hits a negative shock (without requiring a large shock to trigger a large downturn). The novel mechanism here is that the rare event inducing a rarely large downturn does not need to be a rarely large shock, but can simply be a rarely long expansionary period preceding a negative shock of given, fixed size.

Together, we have provided empirical and theoretical foundations for a new form of asymmetry and nonstationarity in business cycles.

5.1 Extensions

There are many variations of the model that could be explored in more detail in future work.

5.1.1 Entry and Exit

Although keeping our theoretical results general allows us to demonstrate that the conclusions apply to many settings, we are then agnostic on things like the entry of firms or search decisions by input suppliers (e.g. workers, banks, or other intermediaries). Both could further amplify some of the effects that we have documented. For example, as workers become scarcer as an expansion endures, the available match quality may be further reduced, further reinforcing the fraction of bad matches. Similarly, as more firms enter the market on the margin during an expansion, they would tend to be firms with worse productivity matches overall.

As another example,[27] when considering capital there can be a further amplification since bad matches in terms of loans have an additional multiplier effect associated with money

[27] As pointed out to us by Jochen Mierau.

creation that feeds back into the economy, for instance, stimulating demand and investment, and creating further incentives for taking on bad matches.

5.1.2 Idiosyncratic Shocks

The results we state apply to some shocks in the abstract, rather than being specific about whether they apply to firm-level, industry-level, or might be economy wide. Of course, shocks operate at all levels, and some firms or industries might be experiencing shrinking demand while the overall economy is in a boom (e.g., see Cooper and Haltiwanger (2006)). As a specific firm experiences some idiosyncratic shock to its demand or costs, it may shed some of its bad matches. This does not stop our results from applying at the industry or economy level, as long as boom periods at industry or economy levels lead to generally higher demand for firms on average and decreases the number of them experiencing negative shocks per unit of time.

Nonetheless, the details of how shocks at various levels interact could be interesting to explore, as they could suggest further testable hypotheses looking cross sectionally across industries. For example, different industries will tend to have different exposures to economy-wide shocks and different incidences of idiosyncratic shocks, and that would lead some industries to exhibit the effects we have outlined more substantially than others.

5.1.3 Productivity Trajectories

Another prediction that emerges from our analysis concerns how productivity varies over the duration of an expansion and eventual contraction. As the relative proportion of bad matches increases as an expansion lasts, we get a prediction that productivity should be dropping over time as an economy continues to expand, and then eventually start increasing once it has gone through a contraction and shed some of the bad matches. These predictions could be explored in more detail, both theoretically and empirically. The model could be enriched to include fixed costs of adding units of production and nonlinear adjustment costs (e.g., see Caballero (1999); Caballero and Haltiwanger (1997)), which would further amplify shocks in both directions, resulting in larger accumulations of bad matches in booms, and more dramatic downswings during contractions.

5.1.4 Demand Evolution

We could also explore additional specifications of the evolution of demand so that it becomes driven by the employment state. In the notation of Section 3, s_t, can depend on the employment levels, g_t, b_t. Higher levels of employment (in the partial ordering on g_t, b_t) lead to a higher probability of the high state H. An explanation for this feedback would be that higher employment of both types of matches leads to more income, production, and hence spending. This leads to a definition of the "*overheating*" of an economy.[28] This feedback amplifies our previous results, as if bad matches have been hired and there is lower unemployment the state is more likely to stay high, which increases the hiring of bad matches. As this is a straightforward extension, we omit the details.

References

Acemoglu, D. and D. Autor (2010): "Skills, Tasks and Technologies: Implications for Employment and Earnings," Tech. Rep. 16082, NBER Working Papers.

Adlakha, S., R. Johari, and G. Weintraub (2011): "Equilibria of dynamic games with many players: existence, approximation, and market structure." *mimeo: Stanford University.*

Azariadis, C. (1975): "Implicit Contracts and Underemployment Equilibria," *Journal of Political Economy*, 83, 1183–1202.

Barnichon, R. (2010): "Productivity and unemployment over the business cycle," *Journal of Monetary Economics*, 57, 1013–1025.

Barnichon, R. and A. Figura (2011): "Labor Market Heterogeneities, Matching Efficiency, and the Cyclical Behavior of the Job Finding Rate," Mimeo, Universitat Pompeu Fabra.

Berger, D. (2011): "Countercyclical Restructuring and Jobless Recov," manuscript, Yale University.

Bergin, J. and D. Bernhardt (1992): "Anonymous sequential games with aggregate uncertainty," *Journal of Mathematical Economics*, 21, 543–562.

——— (2008): "Industry Dynamics with Stochastic Demand," *Rand Journal of Economics*, 39, 41–68.

Bodoh-Creed, A. (2012): "Approximation of Large Dynamic Games," *mimeo - Cornell University.*

Brunnermeier, M. K. and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104, 379–421.

[28] See Kaplan and Menzio (2013) for a recent discussion of the feedback between employment and demand, a topic dating to Keynes (1936).

Caballero, R., E. E. and J. Haltiwanger (1997): "Aggregate Employment Dynamics: Build- ing from Microeconomic Evidence," *American Economic Review*, 87, 115–137.

Caballero, R. J. (1999): "Aggregate Investment," in *Handbook of Macroeconomics*, ed. by J. Taylor and M. Woodford, Amsterdam: North Holland.

Caballero, R. J. and M. L. Hammour (1994): "The Cleansing Effect of Recessions," *American Economic Review*, 84, 1350–68.

——— (1996): "On the Timing and Efficiency of Creative Destruction," *The Quarterly Journal of Economics*, 111, 805–52.

Cooper, R. W. and J. C. Haltiwanger (2006): "On the nature of capital adjustment costs," *The Review of Economic Studies*, 73, 611–633.

Davis, S. J. and J. Haltiwanger (1992): "Gross Job Creation, Gross Job Destruction and Employment Reallocation," *Quarterly Journal of Economics*, 107, 819–863.

Diamond, P. A. (1981): "Mobility Costs, Frictional Unemployment, and Efficiency," *Journal of Political Economy*, 89, 798–812.

——— (1982a): "Aggregate Demand Management in Search Equilibrium," *Journal of Political Economy*, 90, 881–94.

——— (1982b): "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies*, 49, 217–27.

Dow, J. and L. Dicks-Mireaux (1958): "The Excess Demand for Labour: A Study of Conditions in Great Britain, 1946-1956," *Oxford Economic Papers*, 10, 1–33.

Farber, H. S. (1999): "Mobility and stability: The dynamics of job change in labor markets," in *Handbook of Labor Economics*, ed. by O. Ashenfelter and D. Card, Elsevier, vol. 3 of *Handbook of Labor Economics*, chap. 37, 2439–2483.

Furlanetto, F. and N. Groshenny (2012): "Matching Efficiencies and Business Cycle Fluctuations," Working Paper 07, Norges Bank.

Gali, J. and J. Hammour (1992): "Long Run Effects of Business Cycles," Papers 92-26, Columbia - Graduate School of Business.

Hildenbrand, W. (1974): *Core and Equilibria of a Large Economy*, Princeton University Press.

Jaimovich, N. and H. E. Siu (2012): "The Trend is the Cycle: Job Polarization and Jobless Recoveries," Mimeo, Duke University.

Jovanovic, B. (1987): "Work, Rest, and Search: Unemployment, Turnover, and the Cycle," *Journal of Labor Economics*, 5, 131–148.

Kaplan, G. and G. Menzio (2013): "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations," NBER Working Papers 18777, National Bureau of Economic Research, Inc.

Keynes, J. M. (1936): *The General Theory of Employment, Interest and Money*, Palgrave MacMillan.

Koendors, K. and R. Rogerson (2005): "Organizational Dynamics and the Business Cycle: A vew on Jobless Recoveries," *Federal Reserve Bank of St. Louis Review*, 87, 555–579.

Lamadon, T. (2014): "Productivity Shocks, Dynamic Contracts and Income Uncertainty," 2014 Meeting Papers 243, Society for Economic Dynamics.

Michaillat, P. (2012): "Do Matching Frictions Explain Unemployment? Not in Bad Times," *American Economic Review*, 102, 1721–50.

Mortensen, D. T. (1982a): "The Matching Process as a Noncooperative Bargaining Game," in *The Economics of Information and Uncertainty*, NBER Chapters, 233–258.

——— (1982b): "Property Rights and Efficiency in Mating, Racing, and Related Games," *American Economic Review*, 72, 968–79.

Mortensen, D. T. and C. A. Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61, 397–415.

——— (1999): "New developments in models of search in the labor market," in *Handbook of Labor Economics*, Elsevier, vol. 3, chap. 39, 2567–2627.

Philippon, T. (2006): "Corporate governance over the business cycle," *Journal of Economic Dynamics and Control*, 30, 2117–2141.

Pickett, S. and P. e. White (1985): *The ecology of natural disturbance and patch dynamics*, Academic Press: San Diego.

Pissarides, C. A. (1984): "Efficient Job Rejection," *Economic Journal*, 94, 97–108.

——— (1985): "Short-run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages," *American Economic Review*, 75, 676–90.

——— (2000): *Equilibrium Unemployment Theory, 2nd Edition*, vol. 1 of *MIT Press Books*, The MIT Press.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *The American Economic Review*, 95, 25–49.

Shumpeter, J. A. (1939): *Business Cycles: A Theoretical, Historitcal, and Statistical Analysis of the Capitalist Process*, New York, McGraw-Hill.

Steel, Z. L., H. D. Safford, and J. H. Viers (2015): "The fire frequency-severity relationship and the legacy of fire suppression in California forests," *Ecosphere*, 6, 1–23.

Stokey, N. L., R. E. J. Lucas, and E. C. Prescott (1989): *Recursive Methods in Economic Dynamics*, Harvard University Press.

Appendix

5.2 Empirical Appendix

In this appendix, we present a variation on the empirical analysis presented in Section 2. In particular, we repeat the analysis using variables measured in differences from expansion averages rather than absolute levels, and presenting results for the full sample (1969-2012) as well as for the shorter 1969-2005 sample.

We also conduct an alternative test of the predictions of the model using firm-level COMPUSTAT data, where we use BEAA industries (rather than geographic areas) as the relevant economic unit.

Further results also using alternative definitions of crises can be found in the Supplementary Appendix (Section 6).

5.2.1 Variables Measured in Differences from Expansion Mean

The new specification corresponds to equation (1) with variables measured in differences from the expansion mean instead of absolute levels. Let $\tilde{e}_{a,y} = e_{a,y} - \bar{e}_{a,y}$. Similarly, $\tilde{\pi}_{a,y}$ and $\tilde{p}_{a,y}$ will denote proprietors' income growth and population growth in year y transformed in differences from their expansion average in area a. Tables A1 and A2 (only 1969-2005) report estimates from the following linear model:

$$\tilde{e}_{a,y} = \alpha + \gamma \times t_{a,y} + \varepsilon_{a,y} \, [+\psi_y] \, [+\phi_s] \, [+\beta_1 \times \tilde{\pi}_{a,y}] \, [+\beta_2 \times \tilde{p}_{a,y}] \, . \tag{10}$$

Table A1: Estimates from equation (10): full sample (1969-2012)

	(1)	(2)	(3)	(4)
	Deviation of job growth from expansion average			
$t_{a,y}$	-0.111***	-0.078***	-0.052***	-0.054***
	(0.025)	(0.019)	(0.016)	(0.019)
$\tilde{\pi}_{a,y}$			-0.006	-0.003
			(0.005)	(0.006)
$\tilde{p}_{a,y}$			0.683***	0.592***
			(0.211)	(0.204)
Constant	-0.190	-0.438***	1.424**	-0.818
	(0.175)	(0.058)	(0.606)	(0.693)
Sample	1969-2012	1969-2012	1969-2012	1969-2012
Year FE	N	Y	Y	Y
State FE	N	N	N	Y
Observations	783	783	783	783
R-squared	0.025	0.427	0.493	0.550

Robust standard errors in parentheses, clustered at the state-year level
*** p<0.01, ** p<0.05, * p<0.1

This specification confirms our findings from Section 2: there is a significant negative effect of expansion duration on the drop in employment at a crisis onset. Interpreting our estimates, 5 additional years of expansion imply, *ceteris paribus*, an additional drop in job growth (from its expansion average) of between 0.26 percent and 0.63 percent, with larger results (for any set of controls) when we exclude the late 2000s from our analysis.

Table A2: Estimates from equation (10): pre-great crisis (1969-2005)

	(1)	(2)	(3)	(4)
	Deviation of job growth from expansion average			
$t_{a,y}$	-0.126***	-0.092***	-0.072***	-0.078***
	(0.031)	(0.022)	(0.021)	(0.025)
$\tilde{\pi}_{a,y}$			-0.003	0.001
			(0.005)	(0.007)
$\tilde{p}_{a,y}$			0.482**	0.398*
			(0.221)	(0.211)
Constant	-0.345*	0.254***	-0.175	3.132***
	(0.191)	(0.044)	(0.182)	(1.107)
Sample	1969-2005	1969-2005	1969-2005	1969-2005
Year FE	N	Y	Y	Y
State FE	N	N	N	Y
Observations	633	633	633	633
R-squared	0.029	0.417	0.449	0.529

Robust standard errors in parentheses, clustered at the state-year level
*** p<0.01, ** p<0.05, * p<0.1

5.3 Equilibrium Existence

We show the existence of an equilibrium σ^* that is continuous in both arguments μ and s. The proof consists of showing the following claims:

- When a continuous strategy $\hat{\sigma}$ is adopted by all firms, the Markov transition $\mathbf{T}^{\hat{\sigma}}$ describing the evolution of (μ, s) is continuous.

- The optimal solution of the dynamic programming problem of the firm, taking $\mathbf{T}^{\hat{\sigma}}$ as given, results in a continuous function σ.

- For any given initial state, the sum of expected future discounted profits as a function of own strategy σ and others' strategy $\hat{\sigma}$ is continuous in the domain of continuous strategies.

- The best reply function in the firm's problem is continuous in the domain of continuous strategies, and admits a fixed point σ^*, constituting an equilibrium.

Lemma **2** *If the strategy $\hat{\sigma}$ adopted by all firms is a continuous function, the solution to the dynamic programming problem of the single firm taking $\hat{\sigma}$ as given is a continuous function σ.*

Proof of Lemma 2:

The state variable of the firm is a tuple $x = (u, \mu, s, e, b, m)$, where (u, μ, s) evolves according to the transition $\mathbf{T}^{\hat{\sigma}}$, $e \in \{0, 1\}$ denotes whether or not the firm is unmatched $(e = 0)$, $b \in \{0, 1\}$ indicates whether or not bankruptcy occurred $(b = 1)$, and m is the input employed by the firm (when $e = 0$ or $b = 1$ m can be picked arbitrarily without any loss of generality). Let $X = [0, 1] \times \mathcal{P}(M) \times D \times \{0, 1\}^2 \times M$ be the state space of the Markovian decision problem faced by the firm.

With an action a being the probability of accepting a potential match with an input, the feasible action set is simply $\mathcal{A}(x) = [0, 1]$ if x is such that $e = b = 0$ (there is an open vacancy and the firm didn't fail), otherwise no hiring decision has to be made so that $\mathcal{A}(x) = \{0\}$.

Let $\mathbf{P}^{\hat{\sigma}} : X \times [0, 1] \to \mathcal{P}(X)$ be the transition mapping state x and action a into the probability measure $\mathbf{P}^{\hat{\sigma}}(x, a)$ over X.

First we argue that the following is true:

Claim 1 *If $\hat{\sigma}$ is continuous, the transition $\mathbf{T}^{\hat{\sigma}} : \mathcal{P}(M) \times D \to \mathcal{P}(M) \times \mathcal{P}(D)$ is continuous. Thus $\mathbf{P}^{\hat{\sigma}}(x, a)$ is continuous.*

Proof. To see this, notice that $\mathbf{T}^{\hat{\sigma}}(u, \mu, s)$ is the product of a deterministic component mapping (u, μ, s) into the new match state u', μ', and an *iid* random component drawing s' in the next period according to the distribution η: abusing notation $\mathbf{T}^{\hat{\sigma}}(u, \mu, s) = (u', \mu', \eta)$ for all (u, μ, s). Letting $\mathbf{T}_1^{\hat{\sigma}}(u, \mu, s) = (u', \mu')$, this is the function whose continuity is necessary and sufficient for the continuity of $\mathbf{T}^{\hat{\sigma}}$. $\mathbf{T}_1^{\hat{\sigma}}(u, \mu, s)$ depends on ϕ and q for crises and exogenous separation, and on $\hat{\sigma}$ (and F) for acquiring an input. With ϕ and $\hat{\sigma}$ continuous by assumption and F non-atomic this is a composition of continuous functions, preserving continuity. With $\mathbf{T}^{\hat{\sigma}}$ continuous, continuity of the full transition $\mathbf{P}^{\hat{\sigma}}$ follows since the transition of e is *iid* according to the separation rate q (or degenerate when $e = 0$), the transition to bankruptcy (probability that $b = 1$ in the next period) when $e = 1$ (and $b = 0$) is $\int_S \phi\left(\mathbf{T}_1^{\hat{\sigma}}(\mu, s), s', m\right) d\eta(s')$ - continuous from what we have just shown, and the transition of m is continuous in the action a since F is non-atomic. This proves the claim. ■

We now consider the dynamic programming problem of the firm, with Bellman equation

$$V(s) = \max_{a \in \mathcal{A}(x)} \tilde{\pi}(x, a) + \delta \int_S V(x') \, d\mathbf{P}^{\hat{\sigma}}(x, a), \tag{11}$$

and the policy function $\sigma(x)$ being the solution to the maximization problem on the right-hand side in (11). Uniqueness of this solution for all s follows from the fact that the hiring cutoff is always unique as discussed in Section 4.2.

The problem in (11) satisfies the conditions in Theorem 9.6 in Stokey et al. (1989), and continuity of σ follows. ▮

From Lemma 2 we can focus on continuous strategies only, denoting the set of continuous strategies mapping X into $[0,1]$ by $\hat{\Sigma}$.

Now, denote with $\Pi(x;\sigma,\hat{\sigma})$ the discounted expected profits from any initial state x of a firm adopting strategy σ, while the remaining firms in the economy adopt strategy $\hat{\sigma}$.

Lemma **3** *The function* $\Pi(x;\cdot,\cdot):\hat{\Sigma}\times\hat{\Sigma}\to\mathbb{R}$ *is continuous for all* $x\in X$.

Proof of Lemma 3:

Letting $\Pi^T(x;\sigma,\hat{\sigma})$ be the expected profits of the $T<\infty$ periods version of the model, for any $\varepsilon>0$ and for any initial state x there exists a large enough T such that $\left|\Pi(x;\sigma,\hat{\sigma})-\Pi^T(x;\sigma,\hat{\sigma})\right|<\frac{\varepsilon}{2}$ (profits are bounded and $\delta<1$). The result can then be proven by showing that for all T $\Pi^T(x;\cdot,\cdot)$ is a continuous function.

Proceeding by induction, for $T=1$ $\Pi^T(x;\sigma,\hat{\sigma})=\tilde{\pi}(x,\sigma(x))$ for all x. This function is continuous since $\sigma\in\hat{\Sigma}$ and $\tilde{\pi}$ is continuous.

Assuming that for all $t<T$, and all $x\in X$, $\Pi^t(x,\cdot,\cdot)$ is continuous, then

$$\Pi^T(x,\sigma,\hat{\sigma})=\Pi^{T-1}(x,\sigma,\hat{\sigma})+\delta\int_X\tilde{\pi}(x',\sigma(x'))\,d\mathbf{W}^T(x,\sigma,\hat{\sigma}),$$

where $\mathbf{W}^T:X\times\hat{\Sigma}^2\to\mathcal{P}(X)$ is the $T-$periods transition from S to $\mathcal{P}(X)$ when the firm's strategy is σ and the other firms' strategy is $\hat{\sigma}$. Formally: $\mathbf{W}^1(x,\sigma,\hat{\sigma})=\mathbf{P}^{\hat{\sigma}}(x,\sigma(x))$, and recursively $\mathbf{W}^T(x,\sigma,\hat{\sigma})=\int_X\mathbf{P}^{\hat{\sigma}}(x',\sigma(s'))\,d\mathbf{W}^{T-1}(x,\sigma,\hat{\sigma})$. Continuity of $\mathbf{P}^{\hat{\sigma}}(\cdot,\cdot)$ was shown in Claim 1, and non-atomicity of F and continuity of ϕ ensure that also $\mathbf{P}^{\cdot}(x,a)$ (seen as a function of $\hat{\sigma}$) is continuous. Therefore the $T-$periods transition $\mathbf{W}^T$ is continuous implying the continuity of $\Pi^T(x,\cdot,\cdot)$ for all T and all s. ▮

From Lemma 2 and 3 we know that, for any s, the objective of the maximization problem faced by the firm,

$$\max_{\sigma\in\hat{\Sigma}}\Pi(s;\sigma,\hat{\sigma}),$$

is continuous in σ and $\hat{\sigma}$, and the solution is always a singleton. By Berge's theorem of the maximum the best reply function $BR:\hat{\Sigma}\to\hat{\Sigma}$ defined as

$$BR(\hat{\sigma})=\arg\max_{\sigma\in\hat{\Sigma}}\Pi(s;\sigma,\hat{\sigma})$$

is continuous. Since $\hat{\Sigma}$ is compact and convex, Brouwer's fixed point theorem applies and an equilibrium strategy σ^* satisfying $\sigma^*=BR(\sigma^*)$ must exist. ▮

5.4 Proofs of Propositions and Omitted Algebra

Value Functions Derivation:

We compute the expected discounted profits beginning from a period with no match for a richer model, in which possible matches arrive with probability $\alpha \in (q,1]$. In particular, modify the basic model so that when a firm is unmatched a possible match arrives with probability $\alpha \in (q,1]$.[29] If the firm refuses to acquire the input then the firm earns 0 profit for that period and awaits a new match in the next period, which again arrives with probability α. With $\alpha = 1$ this corresponds to the simpler model from Section 3.1.

There are now three strategies that could be optimal acquisition strategies: $\sigma \in \{\sigma_g, \sigma_{Hb}, \sigma_a\}$, where σ_g means that a firm only accepts good matches and acquires them regardless of the state, and σ_{Hb} means that a firm hires good matches in any state and bad matches only in high states; and σ_a means that a firm hires any match when it is unmatched regardless of the state.

We now show that :

$$V(\sigma_g) = \frac{\alpha v E_s(\pi_{sG})}{(1-\delta)(1-\delta(1-q)(1-\alpha v))} \tag{12}$$

$$V(\sigma_{Hb}) = \frac{\alpha v E_s(\pi_{sG}) + \alpha p(1-v)(E_s(\pi_{sB}) + (1-\delta(1-q))(\pi_{HB} - E_s(\pi_{sB})))}{(1-\delta)(1-\delta(1-q)(1-\alpha(v+p(1-v))))}. \tag{13}$$

$$V(\sigma_a) = \frac{\alpha v E_s(\pi_{sG}) + \alpha(1-v)E_s(\pi_{sB})}{(1-\delta)(1-\delta(1-q)(1-\alpha))} \tag{14}$$

Starting with the case when strategy σ_g is adopted it follows that

$$V(\sigma_g) = \alpha v \left(\sum_{\tau=0}^{\infty} q(1-q)^{\tau} \left(\sum_{s=0}^{\tau} \delta^s E_s(\pi_{sG}) + \delta^{\tau+1} V(\sigma_g) \right) \right) + (1-\alpha v)\delta V(\sigma_g),$$

or

$$V(\sigma_g)\left(1 - \delta\left(1 - \alpha v + \frac{q\alpha v}{1-\delta(1-q)}\right)\right) = \frac{\alpha v E_s(\pi_{sG})}{1-\delta(1-q)},$$

which simplifies to

$$V(\sigma_g)(1-\delta(1-q) + \delta\alpha v(1-q)) = \alpha v E_s(\pi_{sG}),$$

or

$$V(\sigma_g)(1-\delta)(1-\delta(1-q)(1-\alpha v)) = \alpha v E_s(\pi_{sG}),$$

yielding the expression in (12).

[29] Requiring $\alpha > q$ ensures that the long-run steady state of the economy with multiple firms (see Section 3.3) features some firms acquiring inputs.

When strategy σ_{Hb} is adopted the value function is instead

$$\begin{aligned}V(\sigma_{Hb}) &= \alpha v\left(\sum_{\tau=0}^{\infty} q(1-q)^{\tau}\left(\sum_{s=0}^{\tau}\delta^{s}E_s(\pi_{sG})+\delta^{\tau+1}V(\sigma_{Hb})\right)\right)\\ &+\alpha(1-v)p\left(\sum_{\tau=0}^{\infty} q(1-q)^{\tau}\left(\pi_{HB}+\sum_{s=1}^{\tau}\delta^{s}E_s(\pi_{sG})+\delta^{\tau+1}V(\sigma_{Hb})\right)\right)\\ &+(1-\alpha(v+p(1-v)))\,\delta V(\sigma_{Hb}),\end{aligned}$$

or

$$\begin{aligned}&V(\sigma_{Hb})\left(1-\delta\left(1-\alpha(v+p(1-v))+\frac{q\alpha(v+p(1-v))}{1-\delta(1-q)}\right)\right)\\ &=\frac{\alpha v E_s(\pi_{sG})+\alpha p(1-v)E_s(\pi_{sB})}{1-\delta(1-q)}+\alpha p(1-v)(\pi_{HB}-E_s(\pi_{sB})),\end{aligned}$$

which simplifies to

$$\begin{aligned}&V(\sigma_{Hb})(1-\delta)(1-\delta(1-q)(1-\alpha(v+p(1-v))))\\ &=\alpha v E_s(\pi_{sG})+\alpha p(1-v)(E_s(\pi_{sB})+(1-\delta(1-q))(\pi_{HB}-E_s(\pi_{sB}))),\end{aligned}$$

yielding the expression in (13).

Finally, for the strategy of acquiring good and bad inputs in either state, σ_a, interesting only if $\alpha << 1$, we have

$$V(\sigma_a)=\alpha\left(\sum_{\tau=0}^{\infty} q(1-q)^{\tau}\left(\sum_{s=0}^{\tau}\delta^{s}E_{sb}(\pi_{sb})+\delta^{\tau+1}V(\sigma_a)\right)\right)+(1-\alpha)\,\delta V(\sigma_a),$$

or

$$V(\sigma_a)\left(1-\delta\left(1-\frac{\alpha(1-\delta)(1-q)}{1-\delta(1-q)}\right)\right)=\frac{\alpha E_{sb}(\pi_{sb})}{1-\delta(1-q)},$$

which simplifies to

$$V(\sigma_a)(1-\delta)(1-\delta(1-q)(1-\alpha))=\alpha E_{sb}(\pi_{sb}),$$

yielding the expression in (14).

If $\alpha = 1$, then the expressions in (12) and (13) reduce to (7) and (8), respectively. Moreover, in that case, it is easy to check that $V(\sigma_{Hb}) > V(\sigma_a)$ (acquiring a bad input in a low state is dominated by waiting).

From these expressions, we see that if α is sufficiently low, it is possible to have accepting any match in any state be the optimal strategy (so that $V(\sigma_a)$ is the maximum of the above three expressions).[30]

Moreover, the comparative statics results from Corollary **??** still hold when comparing $V(\sigma_g)$ and $V(\sigma_{Hb})$, and α has the same effect as v: acquiring bad inputs is more convenient in situations where the rate of arrivals of possible matches is lower.

Proof of Proposition 1:

Consider the expression for $V(\sigma_g)$ and $V(\sigma_{Hb})$ from (7) and (8), respectively. Both denominators are positive, so that $V(\sigma_g) \leq V(\sigma_{Hb})$ if and only if

$$\begin{aligned}&vE_s(\pi_{sG})(1-\delta(1-q)(1-v-p(1-v))) \leq \\ &(vE_s(\pi_{sG})+p(1-v)(\delta(1-q)E_s(\pi_{sB})+(1-\delta(1-q))\pi_{HB}))(1-\delta(1-q)(1-v))\end{aligned}$$

or

$$\begin{aligned}&vE_s(\pi_{sG})\delta(1-q)p(1-v) \leq \\ &(p(1-v)(\delta(1-q)E_s(\pi_{sB})+(1-\delta(1-q))\pi_{HB}))(1-\delta(1-q)(1-v))\end{aligned}$$

and dividing both sides by $p(1-v)$ this condition becomes

$$vE_s(\pi_{sG})\delta(1-q) \leq (\delta(1-q)E_s(\pi_{sB})+(1-\delta(1-q))\pi_{HB})(1-\delta(1-q)(1-v))$$

or

$$\frac{vE_s(\pi_{sG})}{(1-\delta(1-q)(1-v))} \leq \frac{\pi_{HB}}{\delta(1-q)} - (\pi_{HB}-E_s(\pi_{sB}))$$

which corresponds to (9). ∎

Proof of Proposition 2:

Let $m_t = U$ denote that the firm is unmatched at the end of period t.

When $V(\sigma_{Hb}) > V(\sigma_g)$, the transition matrix of the Markov process describing the evolution of m_t along periods with back-to-back H state is

$$\mathbf{T}_{HH} = \begin{bmatrix} (1-q)+qv & q(1-v) & 0 \\ qv & (1-q)+q(1-v) & 0 \\ v & (1-v) & 0 \end{bmatrix},$$

where the first row corresponds to the state $m = G$, the second row to $m = B$, and the third row to $m = U$. The τ−fold matrix product of this matrix, $(\mathbf{T}_{HH} \times \mathbf{T}_{HH} \times ... \times \mathbf{T}_{HH})$

[30] This is easily seen by noting that the denominators all converge to $(1-\delta)(1-\delta(1-q))$ as $\alpha \to 0$, and then the ordering on the numerators is a comparison between $vE_s(\pi_{sG}) + (1-v)E_s(\pi_{sB})$ using σ_a, $vE_s(\pi_{sG})$ using σ_g, and $vE_s(\pi_{sG}) + p(1-v)(E_s(\pi_{sB}) + (1-\delta(1-q))(\pi_{HB} - E_s(\pi_{sB})))$. The first expression is larger than either of the others if $\pi_{HB} - E_s(\pi_{sB})$ is not too large.

τ times, $(\mathbf{T}_{HH})^{\tau}$, is:[31]

$$
\begin{aligned}
(\mathbf{T}_{HH})^{\tau} &= \begin{bmatrix} \frac{1}{v}(v-1) & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} (1-q)^{\tau} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -v & v & 0 \\ -v & v-1 & 1 \\ v & 1-v & 0 \end{bmatrix} \\
&= \begin{bmatrix} v+(1-q)^{\tau}(1-v) & 1-v-(1-q)^{\tau}(1-v) & 0 \\ v-v(1-q)^{\tau} & 1-v+v(1-q)^{\tau} & 0 \\ v & 1-v & 0 \end{bmatrix},
\end{aligned}
$$

from which it follows that the probability of transitioning from $m_{t-\tau} = G$ to $m_t = B$ is $(1-v)(1-(1-q)^{\tau})$, strictly increasing in τ. ▮

Proof of Proposition 3:

We measure the aggregate states of matchings at the end of each period and consider the richer model with random arrival of possible matches at rate α (this covers the $\alpha = 1$ case).

The transition from g_t, b_t to g_{t+1}, b_{t+1} when the state is H is as follows:

$$
g_{t+1} = \underbrace{(1-q)\,g_t}_{\textit{old}\text{ good matches}} + \underbrace{v\alpha\,(q\,(b_t+g_t)+u_t)}_{\textit{new}\text{ good matches}} \tag{15}
$$

$$
b_{t+1} = \underbrace{(1-q)\,b_t}_{\textit{old}\text{ bad matches}} + \underbrace{(1-v)\,\alpha\,(q\,(b_t+g_t)+u_t)}_{\textit{new}\text{ bad matches}} \tag{16}
$$

$$
u_{t+1} = 1 - g_{t+1} - b_{t+1} = (1-\alpha)\,(q\,(b_t+g_t)+u_t)\,. \tag{17}
$$

The system of difference equations (15), (16), and (17) admits a unique globally stable steady state:

$$
g^{SS} = \frac{\alpha v}{1-(1-\alpha)(1-q)}, \tag{18}
$$

$$
b^{SS} = \frac{\alpha(1-v)}{1-(1-\alpha)(1-q)}, \tag{19}
$$

$$
u^{SS} = \frac{(1-\alpha)\,q}{1-(1-\alpha)(1-q)}. \tag{20}
$$

(a) With $s = H$ the economy tends toward full matching ($b^{SS} + g^{SS} = \frac{\alpha}{1-(1-\alpha)(1-q)}$, or

[31] The spectral decomposition of $\mathbf{T}_{HH}$ is:

$$
\begin{bmatrix} \frac{1}{v}(v-1) & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1-q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -v & v & 0 \\ -v & v-1 & 1 \\ v & 1-v & 0 \end{bmatrix}.
$$

To compute the τ-fold matrix product we simply need to take the τ^{th} power of the eigenvalues in the characteristic matrix.

$b^{SS} + g^{SS} = 1$ when $\alpha = 1$). This is the maximum level that can be reached and will never be crossed. Therefore $b_t + g_t$ is (weakly) increasing along sequences of high states.

(b) Clearly since $b_0 < b_{SS}$, $b_t < b_{SS}$ will always be the case (b_t approaches b_{SS} from below, then drops during periods of low states, etc.). Therefore b_t is increasing along periods of back-to-back high states.

From (18) and (19) we have $\frac{b^{SS}}{b^{SS}+g^{SS}} = (1-v)$. Since we assume that $\frac{b_t}{g_t} < \frac{1-v}{v}$ (or equivalently $\frac{b_t}{b_t+g_t} < 1-v$) when the sequence of high states started, this ratio is growing along the sequence, approaching the steady state from below.

(c) The number of firms that would fail if the next period has $s_{t+1} = L$ is

$$b_t \phi \left(\frac{b_t}{b_t + g_t} \right),$$

this number increases since in (b) we showed that both b_t and $\frac{b_t}{b_t+g_t}$ increase and ϕ is an increasing function.

Analogously, the fraction of firms that will fail if the state transitions to low in the next period is

$$\frac{b_t}{b_t + g_t} \phi \left(\frac{b_t}{b_t + g_t} \right), \tag{21}$$

also increasing for the same reasoning as in (c). ∎

In addition to what happens at the onset of a crisis, we can also deduce some dynamics within crises (consecutive periods with low states).

Proposition **6** *Consider a crisis (consecutive sequence of low states):*

(d) *the longer the crisis lasts, the lower the number and the fraction of firms that fail per period,*

(e) *the longer the crisis lasts, the higher both the fraction and mass of good matches in the economy.*

To see the intuition behind (d) and (e), note that there are two forces increasing the fraction of matches that are good, $g/(g+b)$. The first is the elimination of bad matches through accumulated failures, and second is that that vacancies are only filled with good matches in low states. This leads to a decreasing failure rate as only the strong survive, which also leads to fewer failures of remaining bad matches.

Proof of Proposition 6:

The transition from g_t, b_t to L, g_{t+1}, b_{t+1} is described by the following system:

$$g_{t+1} = \underbrace{(1-q)\,g_t}_{\textit{old}\text{ good matches}} + \underbrace{v\alpha\left(q\left(b_t\left(1-\phi\left(\frac{b_t}{b_t+g_t}\right)\right)+g_t\right)+u_t\right)}_{\textit{new}\text{ good matches}} \tag{22}$$

$$b_{t+1} = \underbrace{(1-q)\,b_t\left(1-\phi\left(\frac{b_t}{b_t+g_t}\right)\right)}_{\textit{old}\text{ bad matches in non-failed firms}} \tag{23}$$

$$u_{t+1} = \underbrace{b_t\phi\left(\frac{b_t}{b_t+g_t}\right)}_{\text{workers from failed firms}} + \underbrace{(1-v\alpha)\left(q\left(b_t\left(1-\phi\left(\frac{b_t}{b_t+g_t}\right)\right)+g_t\right)+u_t\right)}_{\text{unmatched workers}}. \tag{24}$$

(d) From (23) it is trivial to see that b_t is decreasing throughout the crisis. To see that

$$\frac{b_{t+1}}{b_{t+1}+g_{t+1}} < \frac{b_t}{b_t+g_t} \tag{25}$$

note that g_t is increasing because as long as g_0 is assumed to be below its maximal steady state level (which is the limit toward which g asymptotes when $s = L$ forever) this must be the case. But then with $b_{t+1} < b_t$ and $g_{t+1} > g_t$ (25) must hold. Recalling that the number of firms that fail in each period is $b_t\phi\left(\frac{b_t}{b_t+g_t}\right)$, the above implies that this is decreasing along the sequence of low states. The same reasoning applies to the fraction of failing firms, as in (21).

The above reasoning also implies (e). ∎

Proof of Proposition 5:

Note that $M_t^{\sigma^*}(u_0, \mu_0, \mathbf{s})$ is defined recursively using the one-period transition function $\mathbf{T}^{\sigma^*}$:

$$M_1^{\sigma^*}(u_0,\mu_0,s_1) = \mathbf{T}^{\sigma^*}(u_0,\mu_0,s_1)\,, \text{ and}$$
$$M_t^{\sigma^*}(u_0,\mu_0,\mathbf{s}) = \mathbf{T}^{\sigma^*}\left(M_{t-1}^{\sigma^*}(u_0,\mu_0,(s_1,...,s_{t-1}))\,,s_t\right).$$

Monotonicity of σ^* and ϕ in s imply that $M_1^{\sigma^*}(u_0,\mu_0,s_1')$ dominates $M_1^{\sigma^*}(u_0,\mu_0,s_1)$, since a higher state induces less layoffs and *worse* acquisitions, thus implying $u_1 < u_1'$ and μ_1 to be first order stochastically dominated by μ_1'.

By the same reasoning, if $M_{t-1}^{\sigma^*}\left(u_0,\mu_0,\left(s_1',...,s_{t-1}'\right)\right)$ dominates $M_{t-1}^{\sigma^*}(u_0,\mu_0,(s_1,...,s_{t-1}))$, no reversals and monotonicity of σ^* and ϕ imply that $M_t^{\sigma^*}(u_0,\mu_0,\mathbf{s}')$ dominates $M_t^{\sigma^*}(u_1,\mu_1,\mathbf{s})$. By induction the statement holds. ∎

Proof of Corollary 2:

Since ϕ is non-increasing in μ_t (in the first order stochastic dominance sense) and m, and μ_t' dominates μ_t, we have the following chain of inequalities:

$$\int_M \phi\left(\mu_t', s_{t+1}, m\right) d\mu_t'\left(m\right) \leq \int \phi\left(\mu_t, s_{t+1}, m\right) d\mu_t'\left(m\right) < \int \phi\left(\mu_t, s_{t+1}, m\right) d\mu_t\left(m\right),$$

where the first quantity on the left is the fraction of firms going bankrupt after $(\mathbf{s}', s_{t+1})$ with starting match state u_1, μ_1, while the last on the right is the same quantity following $(\mathbf{s}, s_{t+1})$ with the same initial state. ∎

Proof of Proposition 4:

From the inductive construction of $M_t^{\sigma^*}$ in the proof of Proposition 5 it follows that for any sequence of states and for all t, $s_t \geq s_{t-1}$ implies that $M_{t-1}^{\sigma^*}\left(u_0, \mu_0, (s_1, ..., s_{t-1})\right)$ dominates $M_t^{\sigma^*}\left(u_0, \mu_0, (s_1, ..., s_t)\right) = \mathbf{T}^{\sigma^*}\left(M_{t-1}^{\sigma^*}\left(u_0, \mu_0, (s_1, ..., s_{t-1})\right), s_t\right)$. This again follows applying no reversal and monotonicity of σ^* and ϕ iteratively. Then

$$EM_t^{\sigma^*}\left(u_0, \mu_0, s_1\right) = E\left[M_t^{\sigma^*}\left(u_0, \mu_0, \mathbf{s}\right) | \mathbf{s} \in G_t\left(s_1\right)\right]$$

is dominated by

$$E\left[M_{t-1}^{\sigma^*}\left(u_0, \mu_0, (s_1, ..., s_{t-1})\right) | \mathbf{s} \in G_t\left(s_1\right)\right] = E\left[M_{t-1}^{\sigma^*}\left(u_0, \mu_0, \mathbf{s}'\right) | \mathbf{s}' \in G_{t-1}\left(s_1\right)\right]$$

which is equal to $EM_{t-1}^{\sigma^*}\left(u_0, \mu_0, s_1\right)$. ∎

6 Supplementary Appendix

In this supplementary appendix to the paper *A Forest Fire Theory of the Duration of a Boom and the Size of the Subsequent Bust*, we investigate the relationship between expansion duration and drop in employment growth using alternative ways of defining a crisis at the BEAA level.

The first alternative that we consider is to use the employment series directly in defining a crisis instead of income levels: a 'crisis' is defined to occur when job growth becomes negative following one or more years of positive growth. This is an interesting case to explore even though it induces a selection biases (since the data involves selection on the dependent variable).

The second alternative that we consider is to define crises still based on income levels, but via different cutoffs of proprietors' income growth: first, we require larger negative drops (with variations from 1 to 10 percent) for a crisis to occur, while in the paper we only required a negative value; second, we define crises based on a certain percentage drop from the previous year's growth rate (intuitively, large drops in the growth rate can identify a period of economic turmoil).

Finally, we report some variations on empirical results from Section 4 in the paper, in which we include one-year expansion observations.

6.1 Defining crises using employment series

We say that area a is at the onset of a crisis in year y if job growth is negative ($e_{a,y} < 0$) following one or more years of positive job growth ($e_{a,y-1} > 0$). Through this we identify 715 crises, of which 542 in the limited sample spanning the 1969-2005 period.

In Tables S1 and S2 we report estimates from the following OLS regression:

$$\tilde{e}_{a,y} = \alpha + \gamma \times t_{a,y} + \varepsilon_{a,y} \, [+\psi_y] \, [+\phi_s] \, [+\beta_1 \times \tilde{\pi}_{a,y}] \, [+\beta_2 \times \tilde{p}_{a,y}] \, .$$

where as in Appendix 6.3.1 of the paper $\tilde{e}_{a,y}$ denotes the difference between job growth at the onset of the crisis and the expansion average; and $\tilde{\pi}_{a,y}$ and $\tilde{p}_{a,y}$, are defined analogously with respect proprietors' income and population, respectively.

When we do not include year fixed-effects the estimated coefficient of interest $\hat{\gamma}$ is not significant, whereas in all specifications with year fixed-effects we obtain significant effects of magnitudes very similar to the one found in the main specification adopted in the paper. This means that, controlling for aggregate year-specific shocks, an area entering a crisis after a longer duration experiences a larger drop in job growth from its expansion mean.

Table S1: Estimates when using employment series to define crises: (1969-2012)

	(1)	(2)	(3)	(4)
	Deviation of job growth from expansion average			
$t_{a,y}$	-0.015	-0.074***	-0.063***	-0.043**
	(0.016)	(0.019)	(0.018)	(0.019)
$\tilde{\pi}_{a,y}$			0.001	-0.000
			(0.005)	(0.005)
$\tilde{p}_{a,y}$			0.465***	0.281**
			(0.108)	(0.133)
Constant	-3.900***	-5.811***	-5.142***	-5.058***
	(0.170)	(2.173)	(1.519)	(1.443)
Sample	1969-2012	1969-2012	1969-2012	1969-2012
Year FE	N	Y	Y	Y
State FE	N	N	N	Y
Observations	715	715	715	715
R-squared	0.001	0.390	0.414	0.554

Robust standard errors in parentheses, clustered at the state-year level
*** p<0.01, ** p<0.05, * p<0.1

Table S2: Estimates when using employment series to define crises: (1969-2005)

	(1)	(2)	(3)	(4)
	Deviation of job growth from expansion average			
$t_{a,y}$	0.029	-0.071***	-0.064***	-0.052**
	(0.019)	(0.018)	(0.018)	(0.022)
$\tilde{\pi}_{a,y}$			-0.003	0.001
			(0.005)	(0.007)
$\tilde{p}_{a,y}$			-0.001	0.000
			(0.006)	(0.005)
Constant	-4.362***	-5.814***	-5.268***	-4.710***
	(0.201)	(2.181)	(1.671)	(1.555)
Sample	1969-2005	1969-2005	1969-2005	1969-2005
Year FE	N	Y	Y	Y
State FE	N	N	N	Y
Observations	542	542	542	542
R-squared	0.004	0.390	0.406	0.534

Robust standard errors in parentheses, clustered at the state-year level
*** p<0.01, ** p<0.05, * p<0.1

6.2 Alternative definition of crises via proprietors' income

6.2.1 Different cutoff values

A more conservative definition of *crisis* than the one adopted in the paper requires the growth of proprietors' income to be below a given negative value (not simply 0), say -0.1 percent, -0.5 percent, etc.

Here we consider defining crises in this alternative way, for 20 different cutoff values of drops in employment between 0 (corresponding to the baseline in the paper) and 2%. In Table S3 below we report the corresponding estimates of γ when running on these datasets the main specification of the paper (column (1) of Table 3, corresponding to equation (7) when controlling for the expansion's trend in job growth and year fixed effects). The table also reports the standard errors (clustered at the state-year level), the two-sided p-value, and the number of crises identified using the various cutoffs.

This shows that our results are robust to alternative cutoffs used to identify area-level crises.

Table S3: Estimates of $\hat{\gamma}$ when identifying crises using larger drops in proprietors' income (cutoff)

Cutoff	0	-0.1%	-0.2%	-0.3%	-0.4%	-0.5%	-0.6%	-0.7%	-0.8%	-0.9%	-1%
$\hat{\gamma}$	-0.048	-0.051	-0.052	-0.052	-0.055	-0.054	-0.055	-0.056	-0.056	-0.054	-0.053
St. Err.	0.018	0.019	0.019	0.019	0.019	0.019	0.020	0.020	0.020	0.020	0.021
p-value	0.007	0.006	0.006	0.007	0.005	0.006	0.005	0.006	0.005	0.007	0.011
Obs.	783	770	758	752	737	734	719	710	699	686	679

Cutoff	-1.1%	-1.2%	-1.3%	-1.4%	-1.5%	-1.6%	-1.7%	-1.8%	-1.9%	-2%
$\hat{\gamma}$	-0.051	-0.050	-0.050	-0.050	-0.048	-0.043	-0.042	-0.042	-0.048	-0.049
St. Err.	0.021	0.021	0.021	0.021	0.022	0.021	0.022	0.022	0.024	0.024
p-value	0.014	0.015	0.018	0.021	0.028	0.046	0.052	0.061	0.041	0.038
Obs.	670	663	651	645	633	625	616	607	599	593

Note: The coefficients are estimated with the same specification adopted in Table 3, column (1) in the paper (Section 4). This correspond to equation (7) when including average job growth during the expansion and year fixed-effects. Standard errors are clustered at the state-year level.

6.2.2 Deviations in growth rate

Another alternative that we consider is to define crises via *relative* deviations in proprietors' income growth rate. In this case, we say that an area enters a crises when proprietors' income growth from one year drops by at least x percentage point in the next year. When doing this for drops between 1 and 10 percent we find results that are still consistent with our main analysis presented in the paper.

Table S4 contains results from this alternative analysis: 'cutoff' indicates the one-year drop in proprietors' income growth that identifies a crisis. We report the estimated coefficient

$\hat{\gamma}$ from the main specification used in the paper: equation (7) when controlling for the expansion's trend in job growth and year fixed effects.

Table S4: Estimates of $\hat{\gamma}$ when identifying crises using deviations in proprietors' income growth

Cutoff	-1%	-2%	-3%	-4%	-5%	-6%	-7%	-8%	-9%	-10%
$\hat{\gamma}$	-0.036	-0.038	-0.034	-0.033	-0.034	-0.033	-0.028	-0.024	-0.032	-0.032
St. Err.	0.010	0.011	0.011	0.012	0.012	0.013	0.014	0.015	0.016	0.017
p-value	0.001	0.000	0.002	0.004	0.005	0.011	0.048	0.101	0.047	0.064
Obs.	1892	1774	1680	1572	1450	1343	1216	1110	1022	935

Note: The coefficients are estimated with the same specification adopted in Table 3, column (1) in the paper (Section 4). This correspond to equation (7) when including average job growth during the expansion and year fixed-effects. Standard errors are clustered at the state-year level.

6.3 Inclusion of one-year expansions

In the analysis presented in the paper, we excluded crises for which the expansion duration was measured to be one year. Given the coarse yearly data provided by the BEA, this cases are subjected to a large measurement error, and likely correspond to a period "in between" a crisis, rather than the crisis onset.

For completeness, Tables S5 and S6 below correspond to Table 3 and Table A1 in the paper when including these observations. Although the added noise reduces the significance of the estimated coefficient $\hat{\gamma}$, the magnitudes and sign are in line with our main results and robust throughout the various specifications.

Table S5: Table 3 in the paper when including crises with one-year expansion

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Jobs growth (onset)							
$t_{a,y}$	-0.023	-0.027**	-0.026**	-0.022*	-0.028*	-0.021	-0.022	-0.020
	(0.015)	(0.013)	(0.013)	(0.013)	(0.016)	(0.014)	(0.014)	(0.014)
$\bar{e}_{a,y}$	0.426***	0.142***	0.142***	0.133**	0.355***	0.137**	0.137**	0.131**
	(0.048)	(0.054)	(0.054)	(0.053)	(0.051)	(0.055)	(0.055)	(0.055)
$p_{a,y}$		0.709***	0.710***	0.723***		0.672***	0.671***	0.679***
		(0.087)	(0.088)	(0.088)		(0.105)	(0.105)	(0.106)
$\pi_{a,y}$			-0.004	0.001			0.004	0.006
			(0.007)	(0.008)			(0.008)	(0.008)
$\bar{\pi}_{a,y}$				0.007*				0.004
				(0.004)				(0.004)
Constant	2.090***	1.359***	1.341***	1.252***	2.828***	1.812***	1.827***	1.761***
	(0.409)	(0.330)	(0.325)	(0.300)	(0.667)	(0.603)	(0.608)	(0.602)
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
State FE	N	N	N	N	Y	Y	Y	Y
Obs.	1,192	1,192	1,192	1,192	1,192	1,192	1,192	1,192
R-squared	0.526	0.596	0.596	0.597	0.580	0.625	0.625	0.625

Robust standard errors in parentheses, clustered at the state-year level.
*** p<0.01, ** p<0.05, * p<0.1

Table S6: Table A1 in the paper when including crises with one-year expansion

	(1)	(2)	(3)	(4)
	Deviation of job growth from expansion average			
$t_{a,y}$	-0.068***	-0.066***	-0.061***	-0.070***
	(0.023)	(0.017)	(0.016)	(0.017)
$\tilde{\pi}_{a,y}$			0.003	0.007
			(0.004)	(0.005)
$\tilde{p}_{a,y}$			0.388**	0.335*
			(0.181)	(0.171)
Constant	-0.546***	1.767***	1.663***	2.270***
	(0.153)	(0.295)	(0.429)	(0.459)
Year FE	N	Y	Y	Y
State FE	N	N	N	Y
Observations	1,192	1,192	1,192	1,192
R-squared	0.007	0.394	0.412	0.448

Robust standard errors in parentheses, clustered at the state-year level
*** p<0.01, ** p<0.05, * p<0.1